INSTRUCTION
for the use of
A.W. FABER "CASTELL"
PRECISION CALCULATING RULES

A.W.
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"CASTELL"
PENCIL WORKS, Ltd.
Established 1761
STEIN
near Nuremberg
LONDON AGENCY:
13/14, Camomile Street, E. C. 3

Wtacy 1936

INSTRUCTIONS

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A.W. FABER "CASTELL"

Precision Calculating Rules

BY

HENRY O. COOPER
AUTHOR OF "SLIDE RULE CALCULATIONS."

MANUFACTORY ESTABLISHED 1761.

Prize Medals from all Important International and Industrial Exhibitions.

A.W. FABER

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Printed in Bavaria.

Fourth Edition.

1/570e. 0004 R 345

EXTRACTS FROM PRESS NOTICES.

„The book is easy to understand, and turns the labour of learning to use the slide rule from a difficult science into a fascinating amusement."

The Architectural Review.

„The book will be particularly useful to students, and will also be of interest to engineers, many of whom might make a greater use of the slide rule than they actually do."

Engineering.

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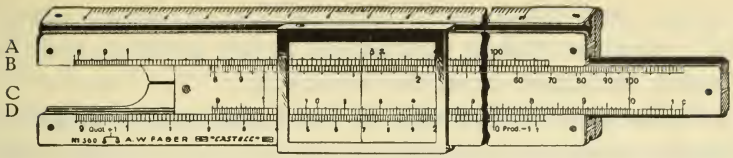


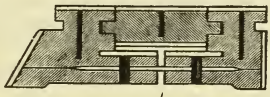
Fig. 1

THE SLIDE RULE.

As a means for carrying out most mathematical operations a well constructed slide rule is an instrument of great utility, saving much labour, and giving the answers to long and complicated problems in a small fraction of the time required to work them in the ordinary way. And, moreover, since its methods are entirely mechanical, the answers will be correct provided the slide rule is properly set and read.

A working knowledge of the slide rule is easily acquired, but practice is necessary to permit of its being used to the best advantage.

Construction of the A.W. FABER "CASTELL" Calculating Rule.



Steel Base

Patent No. 10753

The **A. W. FABER** calculating rule, or slide rule (Fig. 1), consists of a stock, or body, in which moves a strip, called the slide, the face of which is flush with the face of the body.

In construction the body is made in two parts, shaped from well seasoned wood, and reinforced along the centre of each piece by a strip of metal placed edgeways. These metal strips keep the rule perfectly straight; but should it become bent from some accidental cause, the defect may be remedied owing to the presence of this metal.

An even pressure on the slide throughout its length is essential for accurate working; this is obtained in the **A. W. FABER** "CASTELL" Slide Rule by mounting the two portions of the body

upon spring steel plates, which tend to close in against the slide. By this means the slide is held steady in any position, even when it is almost withdrawn from the rule body.

The slide consists of a strip of wood also reinforced by metal in a similar manner to the body of the rule, and arranged with a tongue on each side, which tongues accurately fit the grooves along the inside of the rule body.

This construction of these parts reduces to a minimum the possibility of warping due to atmospheric changes, and enables the rule to give satisfactory service in all climates.

The cursor, which is a piece of glass with one or more fine lines across it carried on an aluminium frame, slides in grooves cut along the outside edge of the body, and may be moved over the length of the rule.

The rule body and slide are faced with dull-finished celluloid, which is held in position by wooden pins. Upon this celluloid division lines have been engraved with mathematical precision. The dark colour given to the lines and numerals causes them to stand out distinctly upon the white celluloid, while the absence of polish upon the surface of the latter minimises glare.

The most common size of slide rule is what is termed the ten inch, although the actual length over all is 11½ inches, and the scales are 9⅞ inches long. Some types are made twenty inches long, as well as pocket sizes as short as six inches and 4¾ inches.

A later form of cursor for the **A.W. FABER** "CASTELL" Slide Rule has the top entirely made of glass which is cemented to guides running in the grooves along the rule body. This cursor gives a clearer view of the scales, and is to be recommended when much calculating has to be done. ⟨Freesight-Cursor⟩.

Cursors carrying magnifying glasses may also be had to fit these rules; by their use accuracy is increased, and calculating generally made easier.

A Brief Explanation of Logarithms.

While, in all probability, the student will be familiar with the principles of logarithms, there may be some who have not studied this branch of mathematics, and although much of the following may be mastered and used successfully by them, the confidence and certainty, coming from a complete understanding of the theories involved, will be lacking. The following description of logarithms is far from a

complete explanation of the subject, but is sufficient to make the principles governing slide rule working perfectly clear.

The logarithm of any number to a given base is the index of the power to which the base must be raised in order to equal the given number. Thus if $A^x = N$, x is called the logarithm of N to the base A. Logarithms to the base 10 are the ones here considered.

The logarithm of 1 is 0; of 10 is 1; of 100 is 2 and so on. Numbers between 1 and 10 must have logarithms between 0 and 1; those between 10 and 100 have logarithms between 1 and 2.

The integral part of a logarithm is called the characteristic, and the fractional part, in the form of a decimal, is called the mantissa.

To obtain the logarithm of a number above 10 and below 100, we add a fraction to the logarithm of 10, that is, we get 1 plus a fraction, and the fraction is called the mantissa. The mantissa is the same for all numbers having the same significant figures and is always positive. For instance, the logarithm of 20 is 1·301, in which 1 is the characteristic, and 0·301 is the mantissa; the logarithm of 200 is 2·301, and of 2 it is 0·301. It will be seen that, in each of these examples, the characteristic alone alters.

In general, the characteristic of a whole number is always 1 less than the number of digits in the number.

Since the logarithm of 1 is 0, the logarithm of 0·1 must be -1, which, to prevent confusion, is written with the minus sign over the characteristic, thus log 0·1 $= \bar{1}$, log 0·01 $= \bar{2}$, log 0·001 $= \bar{3}$, log 0·02 $= \bar{2}$·301, log 0·002 $= \bar{3}$·301.

For a number less than unity the characteristic is 1 more than the number of cyphers immediately following the decimal point and is negative.

The following examples will make this clear: —

EXAMPLE 1. Log 0·055 $= \bar{2}$·7404
Log 0·55 $= \bar{1}$·7404
Log 5·5 $=$ 0·7404
Log 55· $=$ 1·7404
Log 550· $=$ 2·7404

Tables of logarithms give the mantissae of all numbers up to four, and sometimes to seven figures; by means of these tables multiplication, division, involution, and evolution may be performed with ease.

Numbers are multiplied together by adding their logarithms and finding the number corresponding to the logarithm of their product

EXAMPLE 2. Multiply 84 by 26.

Log 84 = 1·9243
Log 26 = 1·4150
3·3393

From a table of logarithms the mantissa is found to give the figures 2184, and the characteristic shows that the number must contain four figures, so the answer is 2184.

For division subtract the logarithm of the divisor from the logarithm of the dividend.

EXAMPLE 3. Divide 20 by 14.

Log 20 = 1·3010
Log 14 = 1·1461
0·1549 = log 1·428

To raise a number to a given power: multiply the logarithm of the number by the index of the power, this gives the logarithm of the number raised to that power.

EXAMPLE 4. Square 320.

Log 320 = 2·5051
Multiplying by 2, we get 5·0102 = log 102,400.

The root of a number is found by dividing the logarithm of the number by the exponent of the root, and finding from tables the number corresponding to the resulting logarithm.

The Slide Rule Scales.

No mention will be made here of any special scales which are peculiar to certain types of slide rule; each type will be described separately in a later section.

The four scales which will be described first are the two on the face of the rule body against the slide, and the two on the slide adjacent to the rule body. (Fig. 1). The edge of the rule which is bevelled will be called the upper edge, and the two scales on that side of the centre will be called the upper scales — one on the rule, and one on the slide — while the two scales on the other side of the centre are the lower scales. Of the upper scales the one on the rule body is called A, while the corresponding one on the slide is B; it is graduated exactly the same as A. The two lower scales, which are exactly alike, are called C and D — C on the slide, and D on the rule body.

Scales A and B are graduated from 0·79 to 128, and scales C and D from 0·89 to 11·2. Each of these scales is really a compact table of logarithms, in which a length corresponds to the logarithm of a number. Referring to scale A, the length from 1 to 2 is proportional to log 2, from 1 to 3 to log 3, and so on. For the sake of clearness the word log is omitted before the numbers on the face of the rule.

It will be remembered that the mantissa of a logarithm is the same for all numbers having the same significant figures irrespective of the position of the decimal point. In the same way the graduations on the slide rule scales give the significant figures only, wirhout any indication of the sizes of the numbers, that being left to the operator to fix from his knowledge of the problem. We may assign any suitable value to 1 on the A scale — called the left hand index of A — such as 10, 100, 1000 etc., or again 0·1, 0·01, 0·001, and, of course, the position of the decimal point will be similarly determined throughout the length of the scale. If the left hand index is taken as 10, then each number along the scale has to be multiplied by 10; or perhaps the left hand index is called 0·1, when each number will be one tenth.

Taking the left hand index of A as 1, there are ten divisions between 1 and 2, each being equal to 0·1. On some rules the figures 1·1, 1·2, 1·3, etc., are marked, while on others the division lines alone are there; this does not make much difference to the ease of reading, since, where the intermediate numbers have been omitted, the line at 1·5 has been extended, and thus stands out, from which the other values are easily found. Each of the ten large spaces between 1 and 2 has been subdivided into five smaller ones, 0·02 each; thus the first mark after 1 is 1·02, then 1·04, 1·06, 1·08, 1·10, 1·12, 1·14, 1·16, 1·18, 1·20, continuing in this order to 2. From 2 we have 2·05, 2·1, 2·15, 2·2, 2·25, 2·3, 2·35, 2·4, 2·45, 2·5 and so on as far as 5, each graduation being greater by 0·05 than the preceding one. From 5 to 10 the increase is 0·1 per division. After 10 the variation is 0·2 per division up to 20, from 20 to 50 it is 0·5, and between 50 and 100 each space represents 1. Some rules have the figures against the division lines 11 to 19, while others just have graduations with the one at 15 extended.

If we now fix the value of the left hand index at 10, the ten divisions from the left hand index are 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, and their sub-divisions are each equal to 0·2, so we get 10, 10·2, 10·4, 10·6, 10·8, 11, etc., to 20, thence 20·5, 21, 21·5, 22,

22·5, 23, etc., the centre of the scale is now 100, and the graduation marked 100 is 1,000. This graduation is called the right hand index of A, and in common with all the indices on the face of the rule and slide, is marked in red. The extensions of the scales to the left of 1 and to the right of 100 call for no extra explanation; their values will be apparent from what has already been said.

Scales A and B are considered as if each were divided at 10, the length from 10 to 100 being called the right hand A or B scale, and from 1 to 10 the left hand scale. It will be understood that the right hand scales have ten times the value of the left hand ones.

Scales C and D extend from 0·89 to 11·2 only, and are consequently more openly spaced than the upper scales. The reading of the lower scales is carried out in the same way as the A and B scales, notice being taken of the fact that the subdivisions are more numerous. The value assigned to the left hand index becomes a multiplier throughout the scale.

Since each length, measured from 1, is proportional to the logarithm of the number at the end of it on all four scales, it will be seen that, as the A and B scales are divided off into spaces half the lengths of those on the C and D scales for the same number, a length on scale D must represent half the logarithm of an equal length on scale A, thus fulfilling the condition necessary for obtaining the square root. For instance, it will be seen that 2 is directly under 4, 3 is under 9, 10 is under 100. In general, every value on A is the square of the value directly beneath it on D.

Before leaving the consideration of the scales on the face of the rule, attention might be called to certain numbers, the locating of which frequently confuses the beginner. Numbers, containing one or two cyphers between the first and second significant figures, are liable to be read as if the cyphers were missing. Numbers such as 20·5 are often taken as 25, or 10·1 as 11. The former is the first graduation after 20 on scale A, and the latter is midway between 10 and the next short division line.

20·75	is half way between the graduations 20·5 and 21.
3·025	is half way between 3 and the first short line after 3.
11·3	is 1½ divisions after 11.
5·75	is 2½ divisions after 5·5.
7·55	is ½ division after 7·5.
101	is ½ division after 10 — taken as 100 — on the right hand A scale, or ½ division after the left hand index — taken as 100 — on the left hand A scale.

The above values should also be found on D.

Much time and trouble will be saved if the graduations on the scales are studied and understood thoroughly before any calculating is done, as only familiarity with the scales will make quick and accurate work possible.

In reading or setting values which do not fall exactly upon division lines the cursor will be found of great assistance, as with it we may easily find the half of any of the small spaces, and quarters, and even smaller sub-divisions may be found with practice. A certain amount of approximating must be done, and the better the scales are known the more accurate will the results be.

Of the three scales on the back of the rule S and T give angles for sines and tangents, and L gives logarithms.

Most of the rules carry scales of inches and millimetres upon their edges.

Multiplication.

The arrangement of scales B and C, on the slide, between scales A and D, on the rule, permits lengths to be added to, and subtracted from the latter by a movement of the slide, thus carrying out in a most convenient manner the operations of multiplying and dividing.

To multiply 2 by 3, we add log 2 to log 3 and obtain log 6.

$$\begin{array}{rl} \text{Log } 2 = & 0{\cdot}3010 \\ \text{Log } 3 = & \underline{0{\cdot}4771} \\ & \underline{\underline{0{\cdot}7781}} = \text{log } 6. \end{array}$$

The slide rule, in doing the same multiplication, adds two lengths proportional to log 2 and log 3 respectively, and the answer instead of being 0·7781 is read directly as 6. The operation with the slide rule is actually accomplished by bringing the beginning of one length on the slide to the end of the other length on the rule, and reading the result on the rule against the end of the second length on the slide.

Example 5. $2 \times 3 = 6$.

Set the left hand index ⟨1⟩ of scale B exactly in line with 2 on scale A, and move the cursor over 3 on scale B. The answer will be found on scale A under the cursor line.

In using the cursor to indicate a position on one scale, which is in direct line with a point on another scale, the hair line is carefully adjusted until it just hides the division line beneath. A reading which does not fall exactly upon a division line must be estimated as to its distance from the nearest graduation, in order to obtain

the last one or two figures. For correct reading with the cursor it is essential that the rule is held so that the light falls upon it in a line perpendicular to its length, as, if the light comes from the side, as shadow will be cast alongside the graduation on the rule, which then will appear broader, and accurate reading will be impossible.

Example 6. Multiply 4 by 12.

Move the left hand index of B to 4 on A, set the cursor over 12 on B, and read the answer, 48, on A under the cursor line.

Example 7. Multiply 7 by 9.

Set the left hand index of scale B to 7 on scale A, move the cursor over 9 on scale B and read the answer, 63, on scale A under the cursor line.

Example 8. Multiply 20 by 30.

Bringing the right hand index ⟨100⟩ of scale B to 20 on scale A we read 6 on A against 30 on B. This answer, clearly, must be multiplied by 100, making it 600.

In practice it is usually known whether the result sought is between 1 and 10, 10 and 100, or 100 and 1000, etc., frequently an approximate answer may be found by inspection, which will be near enough to give the number of figures in the answer.

Example 8 above could have been written $2 \times 10 \times 3 \times 10 = 2 \times 3 \times 10^2$. The product of 2×3 is found on the slide rule and multiplied by 100. In this way the factors may be kept under 10, and the decimal point is moved the number of places shown by the index of the factor containing the tens.

In multiplication using the upper scales, the answer must be multiplied by 100 each time the right hand index ⟨100⟩ of B is used.

Example 9. $64 \times 86 \times 202 = 1{,}110{,}000$.

This may be changed to $6{\cdot}4 \times 8{\cdot}6 \times 2{\cdot}02 \times 10^4$. Set the left hand index of B to 6·4 on A, and bring the cursor over 8·6 on B; without reading the answer, set the right hand index of B under the cursor line, move the cursor over 2·02 on B, and read 1·11 on A under the cursor line. This must be multiplied by 10^6, since we have 10^4 in the example and the right hand index of B was used once. The answer therefore is 1,110,000.

Example 10. $102 \times 136 \times 0{\cdot}0075 = 104$.

Factorising by tens, we get $1{\cdot}02 \times 1{\cdot}36 \times 7{\cdot}5 \times 10$.

Setting the left hand index of B to 1·02 on A and moving the cursor over 1·36 on B we obtain the product of the first two factors; leaving the cursor, and without reading the answer, the left hand index is again brought under the cursor line, and opposite 7·5 on B, 10·4 will be found on A. The answer is thus 104, since we had a factor of 10.

The answer to the above example could have been estimated as follows: taking 100 in place of 102 and moving the decimal two places to the right in the third factor, we get 136×0·75; this may be taken as $\frac{136\times3}{4} = 102$. From this, the answer is seen to be over 100, and the example may be taken as 102×136×0·0075, but the positions of the decimal points will be ignored as only the significant figures need be taken. Either end of the A and B scales may be used, and the significant figures, 104, in the answer will be always given; the decimal point being put in according to the number of figures in the approximate calculation.

For calculations requiring greater accuracy the C and D scales should be used, the wider spacing enabling an extra figure, sometimes two, to be added to the answer.

When using the lower scales for multiplication the following rule gives the number of figures in the answer:

Rule. Add together the number of figures in the different factors, this is the number of figures in the answer if obtained with the slide projecting to the left; from this 1 must be subtracted each time the slide moves to the right.

$\frac{\text{PROD.}}{-1}$, found on the bottom right hand corner of the slide rule, is a reminder of this rule.

Example 11. 14×16=224.

Set the left hand index of scale C to 14 ⟨1·4⟩ on D, and move the cursor over 16 ⟨1·6⟩ on C and read the figures, 224 ⟨2·24⟩, on D under the cursor line; by the rule given above we must have 2+2−1=3 figures in the answer, since the slide went once to the right. The answer is, therefore, 224.

Example: 12. 25×36=900.

Bringing the left hand index of C to 25 ⟨2·5⟩ on D, we read 9 on D against 36 ⟨3·6⟩ on C, and the number of figures is 2+2−1=3, so the answer is 900.

Any number of factors may be multiplied together without reading any answers until the final product is reached. The number of figures in the answer may be found by the above rule.

EXAMPLE 13. $28 \times 164 \times 39 \times 96 = 17{,}200{,}000$.

Set the left hand index of C to 28 ⟨2·8⟩ on D and move the cursor over 164 ⟨1·64⟩ on C, the slide projects to the right; now bring the right hand index of C under the cursor line and set the cursor over 39 ⟨3·9⟩ on C; bring the right hand index of C to the cursor line and move the cursor over 96 ⟨9·6⟩, when 1·72 is read on D under the cursor line. The number of figures will be $2+3+2+2-1=8$, the slide having projected once to the right. The answer then is 17,200,000.

We have, so far, only considered the multiplication of whole numbers, but decimals are no more difficult.

In counting the figures in a decimal, the cyphers between the decimal point and the first significant figure alone are counted and are negative. That this is so will be clear from the following examples: 12 has two digits, 1·2 has one digit, 0·12 has none, 0·012 has minus one, and 0·0012 has minus two digits.

EXAMPLE 14. $98 \times 0{\cdot}046 = 4{\cdot}51$.

Here 0·046 must be taken on the graduation marked 4·6 and 98 on 9·8, so that we are really multiplying the significant figures in the factors, and finding the significant figures in the answer, leaving the decimal point to be put in later according to the rule given. At the same time, it will do no harm to name the graduations on the scales the same as the factors, this will often make it possible to find the size of the answer without the rule for the number of figures.

Set the right hand index of C to 98 on D and against 0·046 on C read 4·51 on D. The size of this answer may be easily estimated by taking 98 as 100 and moving the decimal point on the second factor to 4·6; or by the rule we have $2+\langle -1\rangle = 1$ figure.

The number of figures, which may be read on the slide rule, depends on the part of the rule used. We may easily find 1235 but 4235 calls for considerable judgment in locating the last figure, and 99.45 must be taken as 99·5 on a ten inch slide rule. In cases like this, when the position of the last figure cannot be estimated it is left out; if it be 5 or over the next figure is increased by 1, while if it be less than 5 it is simply dropped.

EXAMPLE: 15. $0{\cdot}172 \times 0{\cdot}039 = 0.00671$.

Set the left hand index of C to 0·172 ⟨1·72⟩ on scale D, and opposite 0·039 ⟨3·9⟩ on scale C read 6·71 on D. The digits will be $0 + \langle -1 \rangle - 1 = -2$, and the answer is 0·00671.

Examples

16. $1{\cdot}16 \times 7{\cdot}3 = 8{\cdot}47$
17. $20{\cdot}5 \times 118 \times 0{\cdot}79 = 1911.$
18. $0{\cdot}068 \times 0{\cdot}00047 \times 0{\cdot}0086 \times 0{\cdot}017 = 4{,}67 \times 10^{-9}.$
19. $1{\cdot}38 \times 19{\cdot}7 \times 10{\cdot}5 \times 100{\cdot}3 = 28600.$
20. $806 \times 0{\cdot}0000802 \times 715000 = 46200.$
21. $160 \times 98{\cdot}1 \times 0{\cdot}0049 \times 860 = 66100.$
22. $37 \times 26 \times 49 \times 286 = 13{\cdot}48 \times 10^{6}.$
23. $356 \times 2810 \times 0{\cdot}00092 = 920.$
24. $201 \times 608 \times 0{\cdot}000402 = 49{\cdot}1.$
25. $1800 \times 1209 \times 0{\cdot}26 \times 3 = 16{\cdot}95 \times 10^{5}.$
26. $63{\cdot}1 \times 24{\cdot}6 \times 35{\cdot}2 \times 0{\cdot}08 = 4370.$
27. $2{\cdot}28 \times 4{\cdot}79 \times 8{\cdot}32 \times 26{\cdot}3 = 2390.$

Division.

Division is performed by subtracting the logarithm of the divisor from the logarithm of the dividend, the result thus obtained being the logarithm of the quotient. This is done on the slide rule by subtracting a length on the slide from another length on the rule, and is exactly the reverse of multiplication.

Example 28. $\frac{9}{3} = 3.$

Setting 3 on B to 9 on A, we find the answer on A against the left hand index of B.

Example 29. $\frac{86}{25} = 3.44.$

Set 25 on B to 86 on A, and over the left hand index of B read the quotient, 3·44, on A.

When the quotient is found against the right hand index ⟨100⟩ of B the answer must be divided by 100.

Example 30. $\frac{25}{31} = 0{\cdot}806.$

Set 31 on B to 25 on A and against 100 on B read 80·6 on A, dividing by 100 we get 0·806.

Any number may be reduced by factorising by tens to give results readable on scale A.

EXAMPLE 31. $\frac{4690}{358} = 13{\cdot}1.$

This becomes $\frac{4{\cdot}69 \times 10^3}{3{\cdot}58 \times 10^2} = \frac{4{\cdot}69}{3{\cdot}58} \times 10.$

Set 3·58 on B to 4·69 on A and against the left hand index of B read 1·31 on A; this must be multiplied by 10, and the answer is 13·1. This example could have been taken as $\frac{46{\cdot}9}{3{\cdot}58}$, which gives the answer without multiplying by 10.

EXAMPLE 32. $\frac{24}{180 \times 37} = 0{\cdot}0036.$

$$= \frac{2{\cdot}4 \times 10}{1{\cdot}8 \times 3{\cdot}7 \times 10^3}$$

$$= \frac{2{\cdot}4}{1{\cdot}8 \times 3{\cdot}7} \times \frac{1}{10^2}$$

Set 1·8 on B to 2.4 on A, move the cursor over the left hand index of B, and, without noting the reading on A, bring 3·7 under the cursor line and read 36 on A against the right hand index of B. This must be divided by 100 since the right hand B index was used, and again by 100 as shown by the second term in the factorised example. Therefore the answer is 0·0036.

When the divisor contains a number of terms or factors, as in the above example, these should not be multiplied together and divided into the dividend, the shortest and most satisfactory method of doing this kind of division is to divide the first term into the dividend, moving the cursor over the first quotient, and bringing the second term to the first quotient — under the cursor line; the cursor is then set over the second quotient and the third term in the divisor brought under the cursor line. In this way no answer is read before the final quotient is reached, and all that need be remembered, with the upper scales, is the number of times the cursor is put over 100 on B.

EXAMPLE 33. $\frac{17}{24 \times 0{\cdot}75 \times 13 \times 90} = 0{\cdot}000807$

$$= \frac{1{\cdot}7}{7{\cdot}5 \times 2{\cdot}4 \times 1{\cdot}3 \times 9} \times \frac{10}{10^2}$$

Set 7·5 on B to 1·7 on A and move the cursor over 100 on B; bring 2·4 on B under the cursor line and put the cursor over the left hand index of B; bring 1·3 under the cursor line and move the

cursor over the left hand index of B; set 9 on B under the cursor line, when 80·7 will be found on A above 100 on B. The right hand index of B was used twice, so the answer will be

$$\frac{80{\cdot}7}{10\times100\times100}=0{\cdot}000807.$$

Combined multiplication and division may be carried out with few movements of the slide if a multiplication follows a division. In these problems the cursor is moved for multiplication and the slide for division.

EXAMPLE 34. $\dfrac{246\times1{\cdot}4\times81\times23}{0{\cdot}008\times63\times146\times28}=311.$

$$=\frac{2{\cdot}46\times1{\cdot}4\times8{\cdot}1\times2{\cdot}3}{8\times6{\cdot}3\times1{\cdot}46\times2{\cdot}8}\times\frac{10^4}{10}$$

Set 8 on B to 2·46 on A and move the cursor over 100 on B, bring the left hand index under the cursor line, since 1·4 on B is too much to the left, and put the cursor over 1·4 on B, set 6·3 on B under the cursor line, move the cursor over 8·1 on B, bring 1·46 under the cursor, move the cursor over 2·3 on B, bring 2·8 on B under the cursor line and read 31·1 above the left hand index of B The slide projected once to the left in division, and we had 10^3 as a multiplier in the problem. The answer is, therefore, 311.

The upper scales are more convenient for multiplication and division, but calculations worked on the lower scales are usually more exact. The latter, therefore, should be used when greater accuracy is required.

The rule for the number of digits in a quotient obtained on the C and D scales is as follows:

RULE. Subtract the number of figures in the divisor from the number in the dividend; this is the number of figures in the answer when the slide projects to the left. When the slide goes to the right add 1 to this.

⟨QUOT⟩
⟨ + 1 ⟩ on the left hand corner of the slide rule face is a reminder of this rule.

EXAMPLE 35. $\dfrac{1.25}{7{\cdot}5}=0{\cdot}1667.$

Bringing 7·5 on C to 1·25 on D we read 0·1667 on D under the index of C, since the number of figures is $1-1=0$.

EXAMPLE 36. $\frac{0{\cdot}0038}{0{\cdot}062} = 0{\cdot}0613.$

Set 0·062 on C to 0·0038 on D and read 0·0613 on D against the index of C. The digits are $-2 - \langle -1 \rangle = -1$.

EXAMPLE 37. $\frac{1}{960} = 0{\cdot}001042.$

Set 960 on C to 1 on D and read 0·001042 against the index of C. The figures are $1 - 3 = -2$.

EXAMPLE 38. $\frac{945 \times 23}{635} = 34{\cdot}2.$

Set 635 on C to 945 on D and move the cursor over 23 on C. The answer, 34·2, is found under the cursor line on D. The number of figures is $3 + 2 - 3 + 1 - 1 = 2$, since the slide projected to the right once each in multiplication and division.

EXAMPLE 39. $\frac{1{\cdot}08 \times 64 \times 0.0307 \times 19{\cdot}75}{1{\cdot}25 \times 0{\cdot}0016 \times 45 \times 8000} = 0{\cdot}0582.$

By re-arranging the terms in this example, so as to have numbers with, as near as possible, the same significant figures in the same order in the numerator and denominator, the movements of the slide will be lessened. The above example may be written

$$\frac{1{\cdot}08 \times 64 \times 19{\cdot}75 \times 0{\cdot}0307}{1{\cdot}25 \times 45 \times 0{\cdot}0016 \times 8000}.$$

Set 1·25 on C to 1·08 on D, move the cursor over 64 on C, bring 45 on C under the cursor line, move the cursor over 19·75 on C; set 0·0016 under the cursor, put the cursor over 0·0307 on C, and bring 8000 under the cursor line; the significant figures in the answer, 582, are read on D against the right hand index of C. The number of figures will be $4 - 5 = -1$, since the slide projected to the right twice each in multiplication and division, thus cancelling. The answer is 0·0582.

In practical culculations these questions of the position of the decimal point seldom arise. The size of the answer is generally approximately known, and often a rough mental calculation will give a result sufficiently near to allow the number of figures to be fixed without using the rules. Frequently, by moving some of the decimal points without altering the value, a problem may be simplified, when, by roughly cancelling, the approximate size of the answer will be apparent.

Where the foregoing rules for the number of digits are used it is helpful to note the movements of the slide to the right by means of two groups of dots or strokes; one for division and the other for multiplication. These are cancelled between groups whenever possible, and the remainder added to, or subtracted from the difference between the number of digits in the numerator ane denominator terms.

The extensions, which have already been referred to, at the beginning and ends of the scales are designed to avoid changing indices in multiplication and division when a factor, or an answer, lies just beyond the left or right hand index. Care is required in applying the rules for the number of digits in products or quotients on the occasions that these extensions are used, as they have the same values as those graduations at the opposite ends of the scales which bear the same significant figures.

The following examples will provide practice in multiplication and division:

EXAMPLES

40. $\dfrac{102 \times 0{\cdot}14 \times 76{\cdot}3 \times 25{\cdot}4}{760 \times 36{\cdot}5 \times 0{\cdot}0084 \times 23} = 5{\cdot}16.$

41. $\dfrac{0{\cdot}015}{78 \times 16 \times 0{\cdot}0036 \times 1475} = 0{\cdot}000{,}002{,}262.$

42. $\dfrac{16{\cdot}5 \times 23{\cdot}6 \times 24{\cdot}6 \times 6}{3{\cdot}75 \times 2{\cdot}4 \times 192 \times 0{\cdot}063} = 528.$

43. $\dfrac{2240 \times 16 \times 9{\cdot}6}{5460 \times 288 \times 42{\cdot}5} = 0{\cdot}00515.$

44. $\dfrac{24 \times 19500 \times 62 \times 108}{2250 \times 764 \times 826 \times 9} = 0{\cdot}245.$

45. $\dfrac{1}{28{\cdot}5 \times 1{\cdot}04 \times 0{\cdot}006} = 5{\cdot}63.$

46. $\dfrac{0{\cdot}00435}{1{\cdot}4 \times 7{\cdot}63 \times 0{\cdot}00916} = 0{\cdot}0445.$

Proportion.

The ease with which any ratio may be set up between the scales on the slide rule makes it very useful for the solution of problems in proportion. Either upper or lower scales may be used for this purpose, depending upon the accuracy required.

When 1 on C is set to 2 on D, a ratio of 2 to 1 is established between these two scales. By setting 1 on C to any number x on D, we obtain the ratio of x to 1 throughout the length of scales.

The fourth term of a proportion may be found by setting the first term on C against the second term on D, and opposite the third term on C will be found the fourth term on D.

Example 47. 19 : 25 : : 30·4 : x.

Set 19 on C to 25 on D, and opposite 30·4 on C read 40 on D.

Any term of a proportion may be obtained in this way.

Example 48. 18 : 42 : : x : 70.

Set 18 on C to 42 on D, and opposite 70 on D read 30 on C.

The slide rule forms a convenient means of converting from one system of units to that of another. Once the ratio between the two systems is established any value may be read off without re-setting the rule.

Example 49. Determine the number of centimetres in 8 inches.

Since 1 inch equals 2·54 centimetres, set 10 inches on C to 25·4 centimetres on D, and against 8 inches on C read 20·3 centimetres on D.

On the back of the slide rule will be found many ratios, which are very useful for finding the equivalent values in different systems of units. For inches to millimetres the ratio given is 5 on C to 127 on D, which means that 5 ins = 127 mm.

Example 50. Find the weight in pounds of 25 cubic feet of water.

Set 17 cubic feet on C to 1060 pounds on D, and against 25 cubic feet on C will be found 1560 pounds on D.

Example 51. Convert 1620 pounds to kilogrammes.

Set 280 pounds on C to 127 kilogrammes on D, put the cursor over the right hand index of C and bring the left hand index under the cursor line ‹this is necessary since 1620 pounds on C is beyond the D scale›, and against 1620 pounds on C read 735 kilogrammes on D.

Example 52. Find the pressure, in pounds per square inch, of 4 atmospheres.

Set 970 pounds on C to 66 atmospheres on D and over 4 atmospheres on D read 58·8 pounds on C.

Example 53. Convert 700 miles to kilometres.

Set 87 miles on C to 140 kilometres on D and under 700 miles on C read 1126 kilometres on D.

If the numerator of a fraction on C is set against the denominator on D, the decimal equivalent of the fraction will be found on C against the index of D.

EXAMPLE 54. Reduce $\frac{3}{16}$ to a decimal.

Set 3 on C to 16 on D and above the left hand index of D read 1·875 on C, as this is over 10 on D the answer is $\frac{1{\cdot}875}{10} = 0{\cdot}1875$. It will be clear that the left hand index of D will be 10, since 1·6 is taken as 16.

EXAMPLE 55. Find the decimal equivalent of $\frac{2}{25}$.

Bring 2 on C to 25 on D and read $\frac{8 \text{ on C}}{100 \text{ on D}} = 0{\cdot}08$.

To find the vulgar fraction from a decimal, put the decimal on C over the index of D, and above the required denominator on D read the numerator on C.

EXAMPLE 56. Convert 0·625 to eighths.

Set $\frac{6{\cdot}25 \text{ on C}}{10 \text{ on D}}$ and over 8 on D read the numerator 5; the fraction, therefore, is $\frac{5}{8}$.

Percentages are worked in the same way as proportion, but the indices, 1 or 10, on the lower scales are taken as 100%.

To find 7% of 120;

set $\frac{100\% \langle 1 \rangle \text{ on C}}{120 \text{ on D}}$ and read $\frac{7\% \text{ on C}}{8{\cdot}4 \langle \text{the answer} \rangle \text{ on D}}$.

To determine what percentage 8·4 is of 120;

set $\frac{100\% \langle 1 \rangle \text{ on C}}{120 \text{ on D}}$ and read $\frac{7\% \langle \text{the answer} \rangle \text{ on C}}{8{\cdot}4 \text{ on D}}$.

To find the number of which 8·4 is 7%;

set $\frac{7\% \text{ on C}}{8{\cdot}4 \text{ on D}}$ and read $\frac{100\% \langle 1 \rangle \text{ on C}}{120 \langle \text{the answer} \rangle \text{ on D}}$.

It will be seen that the settings are exactly the same in the three foregoing problems. As this method of setting is employed in all percentage problems, it will be found profitable to consider these carefully.

To increase 318 by 5%;

set $\frac{100\% \langle 1 \rangle \text{ on C}}{105\% \langle 100+5 \rangle \text{ on D}}$ and read $\frac{318 \text{ on C}}{334 \langle \text{the answer} \rangle \text{ on D}}$.

The 105% in the last setting is taken near the left hand index.

To decrease 210 by 10%

set $\frac{90\% \langle 100-10 \rangle \text{ on C}}{100\% \langle 10 \rangle \text{ on D}}$ and read $\frac{189 \langle \text{the answer} \rangle \text{ on C}}{210 \text{ on D}}$.

The 90% in the above setting is close to the right hand index.

Reciprocals.

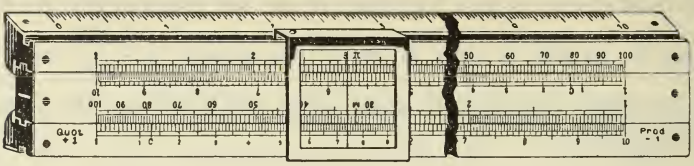


Fig. 2

If the slide be removed from the rule and replaced with scale C against scale A and B adjacent to D (in this position the numbers will be upside down), Fig. 2, the reciprocals of the numbers on D may be read on the C scale when the indices of the four scales coincide at each end.

Scales C and B, when thus inverted, will be called CR and BR.

In reading reciprocals the location of the decimal point is simply determined. The rule is the same as for division on the lower scales, but since the slide remains stationery, the numher of cyphers following the decimal point in the reciprocal is always 1 less than the number of figures in the number when it is an integer; with a decimal the number of figures in the reciprocal is 1 more than the number of cyphers immediately following the decimal point in the number.

Example 57. Determine the reciprocals of 5, 3, 26, 0·5, 0·25, 0·002.

With the slide inverted and the indices at both ends in line, put the cursor over 5 on D and read 0.2 on CR; above 3 on D read 0·333 on CR; over 26 on D read 0·0385 on CR; over 0·5 read 2; over 0·25 read 4; and over 0·002 read 500.

With the inverted slide multiplication and division may be performed, but since on CR we are dealing with reciprocals, it follows that the ordinary methods of multiplying and dividing, with the slide normal, will be reversed. For instance, to divide 25 by 4, we set the left hand index of CR to 25 on D, and under 4 on CR read 6·25 on D. To multiply 82 by 3 we set 3 on CR to 82 on D and read 246 on D against the left hand index of CR. It will be understood that these two examples are $25 \times \frac{1}{4}$ and $\frac{82}{1/3}$.

Further consideration of the slide rule with the slide inverted discloses another fact which occasionally proves very useful. With the slide set so that the right hand index of CR is in line with 10 on D, we notice that as the cursor is moved over any number on D, so that number multiplied by the number on CR, under the cursor,

always gives a product of 10, while between BR and A we have a constant product of 100. This is so for any number to which the index of BR or CR is set on scales A or D. For instance, if the right hand index of CR is set opposite 84 on D, then opposite 80, 70, 60, 40 and 20 on D will be found 1·05, 1·2, 1·4, 2·1 and 4·2 on CR, as well as any intermediate number required. This will be found useful in the calculation of surfaces where the length and breadth vary, but the area remains constant.

Example 58. Compare the length of flooring required to cover 2400 square feet with boards 5, 6 and 7 inches wide.

First find the decimal equivalent of a foot of the above widths by setting 1 on C to 12 on D, with the slide normal, then against 5, 6 and 7 inches read 0·416, 0·5 and 0·584 feet respectively on C. Now invert the slide, and set the left hand index of CR against 2400 on D and, moving the slide over the width of boards 0·416, 0·5, and 0·584 feet on CR, read the lengths 5770, 4800, and 4110 feet on D.

Squares.

Mention has already been made of the connection between the upper and lower scales, whereby each value on scale A or B is the square of the value directly beneath it on D or C. This arrangement allows of squares and square roots being read off without moving the slide.

The square of any number is found by placing the cursor over the number on D, and reading the square on A under the cursor line. The following rule gives the number of digits in the square:

Rule. The square contains twice the number of figures in the number when read on the right hand A scale; when read on the left hand A scale, the square has 1 figure less than this.

Example 59. $35^2 = 1225$.

Set the cursor over 35 on D and read 122 on A, since this is read on the right hand A scale the digits will be $2 \times 2 = 4$. In this case it is impossible to read nearer than 1220 on the slide rule, but it is seen that the square is just over 1220, and from the number it is obvious that the last figures must be 5. Therefore the answer is 1225.

Example 60. $134^2 = 17950$.

The number of figures in the square is $3 \times 2 - 1 = 5$, since the answer was read on the left hand A scale. It is again impossible to read the fourth figure, but the reading is between 17900 and 18000

and the answer may be taken half way between these two; any intermediate number would make quite a negligible error. The exact square of 134 is 17956.

EXAMPLE 61. $1{\cdot}5^2 = 2{\cdot}25$.

Since this square is found on the left hand A scale the number of figures is $1 \times 2 - 1 = 1$.

EXAMPLE 62. $4{\cdot}6^2 = 21{\cdot}16$.

Here the right hand A scale is used, and the number of digits is $1 \times 2 = 2$.

The squares of numbers wholly decimal are found in the same way.

EXAMPLE 63. $0{\cdot}2^2 = 0{\cdot}04$.

The left hand A scale is used, and the number of figures is $0 \times 2 - 1 = -1$, that is, one cypher after the decimal point.

EXAMPLE 64. $0{\cdot}4^2 = 0{\cdot}16$.

The number of figures is $0 \times 2 = 0$.

EXAMPLE 65. $0{\cdot}0024^2 = 0{\cdot}000{,}005{,}76$.

The number of digits will be $-2 \times 2 - 1 = -5$.

Squares may of course be found by multiplying, but the above method is quicker and more convenient.

Square Roots.

Square root, in spite of its being carried out in the opposite manner to squaring — taking the number on A and obtaining its root on D — requires care in selecting the end of the A scale to be used for numbers above 100 or less than 1. If the number be pointed off in periods of two figures each, as in arithmetic, to the left for a whole number, or to the right for a decimal, no difficulty will be experienced. When the first period on the left contains one figure the left hand A scale is used; when it contains two figures the number must be taken on the right hand A scale. This will be obvious since the left hand period to contain one figure must be less than 10, and have a square root less than $\sqrt{10}$; with two figures, the left hand period will be between 10 and 100 and have a square root between $\sqrt{10}$ and $\sqrt{100}$. In the root there will be one figure for each period in the number. From the foregoing it will be plain that odd numbers of digits are taken on the left hand A scale, and even numbers of digits on the right. Decimals, having no cyphers or an even number of cyphers immediately after the decimal point, are taken on the right hand A scale; while those having an uneven number of cyphers are taken on the left.

If a rule be preferred for fixing the numbers of digits in the square root, the following may be used:

Rule. Calling the number of figures in the number N, and in the root n. Then, when N is an even number, $n = \frac{N}{2}$, and the right hand A scale is used. When N is odd, $n = \frac{N+1}{2}$, and the left hand A scale is required. With this rule the numbers, of course, need not be pointed off.

Example 66. $\sqrt{120} = 10{\cdot}95$.

This is taken on the left hand A scale, since the number of digits is odd and, when pointed off from the decimal point, there are two periods, requiring two figures in the answer; or using the rule given $N = 3$, and $n = \frac{3+1}{2} = 2$ figures.

Example 67. $\sqrt{7200} = 84{\cdot}9$.

This being an even number of figures the right hand A scale is used. Setting the cursor over 72 on A we read 8·49 on D, and, since two figures are required, the root is 84·9.

Example 68. $\sqrt{0{\cdot}016} = 0{\cdot}1265$.

Setting off two places from the decimal point we obtain 0·01,60, the root of the first period must be 0·1, so this is taken on the left hand A scale. Bringing the cursor to 1·6 on A we read 1·265 on D, and the answer is 0·1265. By the rule $N = -1$, and $n = \frac{-1+1}{2} = 0$.

Example 69. $\sqrt{19300} = 139$.

Pointing this off we see that 1 is the root of the first period on the left, so the left hand A scale must be used, and the answer contains three figures.

Square roots may be extracted by using the inverted slide, and working on scales CR and D. The right or left hand index of CR is set against the number on D, depending on whether it contains an odd or even number of digits, and the cursor is moved along until the same number appears on CR and D simultaneously under the cursor line. This number is the required root. This method usually gives more accurate results, since the two lower scales are used. The reason for this way of finding square roots will be apparent from consideration of multiplication with the inverted slide.

Cubes.

For cubes and cube roots the A, D and left hand B scales are used, the latter being given the same value as the D scale.

A number is raised to the third power by combining the operation of squaring with that of multiplication; the cursor is set over the number on D and the index of B is brought under the cursor line, when against the number on the left hand B scale will be found the cube on A.

Three cases may occur and are known from the position of the slide and the portion of the A scale upon which the answer is found; from these particulars the number of figures in the cube is determined.

Case 1. The cube is found on the left hand A scale with the slide projecting to the right, when $N = 3n - 2$.

Case 2. The cube is found on the right hand A scale with the slide projecting to the right, when $N = 3n - 1$.

Case 3. The cube is found on the left hand A scale with the slide projecting to the left, when $N = 3n$.
N and n are the number of digits in the cube and number respectively.

Example 70. $2^3 = 8$.

Set the cursor over 2 on D, bring the left hand index of B under the cursor line, and against 2 on B read 8 on A. This is case 1, since the slide projects to the right and the answer is read on the left hand A scale, hence the number of figures in the cube will be $3 \times 1 - 2 = 1$.

Example 71. $4^3 = 64$.

Move the cursor over 4 on D, bring the left hand index of B under the cursor line and read the answer, 64, on A against 4 on B. This is case 2, and $N = 3 \times 1 - 1 = 2$.

Example 72. $6^3 = 216$.

Put the cursor over 6 on D and bring the right hand index of B to the cursor line, when the answer, 216, will be found on A against 6 on B. This is case 3, so $N = 3 \times 1 = 3$.

Example 73. $70^3 = 343{,}000$.

The slide projects to the left and the cube is read on the left hand A scale, hence $N = 3 \times 2 = 6$.

Example 74. $0{\cdot}361^3 = 0{\cdot}047$.

This is found on the right hand A scale with the slide to the right, thus $N = 3 \times 0 - 1 = -1$.

EXAMPLE 75. $0{\cdot}0038^3 = 0{\cdot}000{,}000{,}054{,}9$.

The slide projects to the right and the cube is read on the right hand A scale, and $N = 3\ \langle -2 \rangle - 1 = -7$.

EXAMPLE 76. $0{\cdot}078^3 = 0{\cdot}000475$.

The answer is read on the left hand A scale with the slide to the left, $N = 3\ \langle -1 \rangle = -3$.

By writing example 76 in the form $7{\cdot}8^3 \times 10^{-2\times3}$ the answer will be easily found without rules for the number of decimal places. The cube of 7·8 must be above 100 and under 1000.

Therefore $7{\cdot}8^3 \times 10^{-6} = 475 \times 10^{-6} = 0{\cdot}000475$.

This method may be employed for any number, and by shifting the decimal point to leave only one figure to the left of it, the number of digits will be apparent from the size of the integer.

EXAMPLE 77. 9800^3

$$= 9{\cdot}8^3 \times 10^{3\times3}$$
$$= 941 \times 10^9$$
$$= 941{,}000{,}000{,}000.$$

EXAMPLE 78. 1860^3

$$= 1{\cdot}86^3 \times 10^{3\times3}$$
$$= 6{\cdot}43 \times 10^9.$$

EXAMPLE 79. $0{\cdot}00484^3$

$$= 4{\cdot}84^3 \times 10^{-3\times3}$$
$$= 113 \times 10^{-9}.$$

EXAMPLE 80. 121^3

$$= 1{\cdot}21^3 \times 10^{2\times3}$$
$$= 1{\cdot}77 \times 10^6.$$

Cube Roots.

To find the cube root of a number, set the cursor over the number on A and move the slide until the same number is seen simultaneously under the cursor on the left hand B scale and on D opposite the index of B. This number is the required root.

The part of the A scale upon which the number is taken, and the end of the rule at which the slide projects are determined from the number of figures in the number, in the following manner:

Point the number off in periods of three figures from the decimal point; the period on the left may contain one, two or three figures.

Case 1. If the left hand period contains one figure the left hand A scale is used with the slide to the right, and

$$n = \frac{N + 2}{3}.$$

Case 2. If the left hand period contains two figures the right hand A scale is used with the slide to the right, and

$$n = \frac{N + 1}{3}.$$

Case 3. If the left hand period cotains three figures the left hand A scale is used with the slide to the left, and

$$n = \frac{N}{3}.$$

In applying these rules to numbers wholly decimal the period on the left containing the first significant figure must be considered, since if the first period contains three cyphers its root will be 0.

Example 81. $\sqrt[3]{8} = 2.$

This is case 1, so the left hand A scale is used, the slide will project to the right, and $n = \frac{1+2}{3} = 1$. Set the cursor over 8 on A and move the slide slowly to the right until the same number, 2, is under the cursor line on B and on D in line with the left hand index of B or C.

Example 82. $\sqrt[3]{27} = 3.$

This is case 2, requiring the right hand A scale and the slide to the right, also $n = \frac{2+1}{3} = 1$. Put the cursor over 27 on A and move the slide until the same reading is seen on the left hand B scale under the cursor and on D against the left hand index of C. The only number which may be thus read with this setting is 3.

Example 83. $\sqrt[3]{512} = 8.$

This is case 3, so the left hand A scale must be used with the slide to the left, and $n = \frac{3}{3} = 1.$

Example 84. $\sqrt[3]{27000} = 30.$

Pointing this off we get 27,000, hence this is case 2, and, since we require one figure in the root for each period in the number, we find the cube root of 27, and add one cypher; or $n = \frac{5+1}{3} = 2.$

Example 85. $\sqrt[3]{0{\cdot}0042} = 0{\cdot}1613.$

When pointed off this becomes 0·004,200, which is case 1, since the first period contains one figure, thus $n = \frac{-2+2}{3} = 0$.

EXAMPLE 86. $\sqrt[3]{0{\cdot}025} = 0{\cdot}292$

This is case 2, and $n = \frac{-1+1}{3} = 0$.

EXAMPLE 87. $\sqrt[3]{0{\cdot}00025} = 0{\cdot}063$.

Pointing off, we obtain 0·000,250; from this it is seen to be case 3, so the left hand A scale is used with the slide to the left, and $n = \frac{-3}{3} = -1$.

From these examples it will be understood that to find the cube root the operations on the slide rule are exactly the reverse of those required to find the cube.

After a whole number has been pointed off in periods of three figures, an approximate value for the cube root of the left hand period may be found by inspection. The largest number possible for this is under $\sqrt[3]{1000} = 10$, and then only when the left hand period contains three figures, in which case the lowest value of the root cannot be less than $\sqrt[3]{100} = 4{\cdot}64$; with two figures in the left hand period the root is between 4·64 and $\sqrt[3]{10} = 2{\cdot}15$; while if there be one figure the cube root will be under 2·15. Having noted the approximate value of the root of the first period a trial on the slide rule will show whether the correct end of A is being used, as, if the wrong end be taken, it will be at once apparent that the approximate root on the D and on the left hand B scales cannot be read simultaneously.

A decimal may be treated in the same way if it be first brought to a whole number by shifting the point three or six places to the right. When the root has been determined the decimal point must be moved to the left one place for each three it was previously moved to the right.

A decimal may be conveniently changed into a whole number for the purpose of finding its cube root by factorising it, taking 10^{-3} or 10^{-6} as one factor.

EXAMPLE 88. $\sqrt[3]{0{\cdot}00416} = 0{\cdot}161$

$$= \sqrt[3]{4{\cdot}16} \times 10^{-\frac{3}{3}}$$

$$= \sqrt[3]{4{\cdot}16} \times 10^{1-}.$$

It will be seen that the cube root of 4·16 must be less than $\sqrt[3]{8} = 2$, and so the left hand A scale will be used with the slide to the right. The cube root of 4·16 is found to be 1·61. This must be multiplied by 10^{-1}, so we get $1{\cdot}61 \times 10^{-1} = 0{\cdot}161$.

From the foregoing it will be apparent that cube roots may easily be found without using rules to determine the portion of the A scale required, or the number of figures in the answer.

Cube roots can also be extracted by means of the reversed slide, using scales C and A ⟨scale C will now be referred to as CR⟩. The calculation is carried out by bringing the index of CR to the number on A, and moving the cursor along until the same number is found on CR and on A under the cursor line. This is the cube root. After the number has been pointed off in periods of three figures: if the left hand period contains one or two figures, use the right hand index, if it contains three figures, use the left hand index. The root is always read on the left hand A scale.

Logarithms.

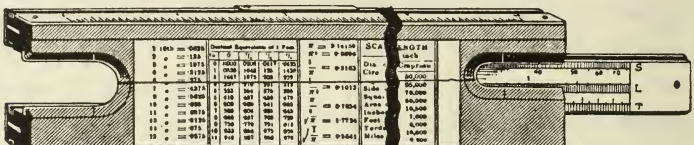


Fig. 3

The evenly divided scale, marked L, Fig. 3, on the back of the slide, used in combination with scale D enables logarithms to be found. It is graduated from 0 to 1, thus giving the logarithms of numbers from 1 to 10, as read on the D scale. Since the characteristics of numbers within these limits are 0, it follows that the L scale gives the mantissa of any number whatsoever, the characteristic being determined in the usual way, as explained on page 7.

Looking at the back of the slide rule it will be noticed that a slot thas been cut in the body at each end, thereby exposing a portion of the scales on the back of the slide. The L scale is read against the small line on the bevelled side of the slot, at the right hand end of the rule. This line, which may be called the L index line, shows the position of the right hand index on the rule face.

Since the numbers on L run from right to left great care is required in reading on this scale. The graduations marked 1, 2, 3, etc. are read as 0·1, 0·2, 0·3 etc., the last line on the extreme left is 1 and is against the L index line when the left hand index of C is against 10 on D, this being log 10. The long line across the right hand end of the three scales is 0 on the L scale and represents log 1. Between this line and 0·1 are ten divisions each equal to 0·01; these ten divisions are again subdivided into five smaller ones 0·002 each. This order is maintained throughout the length of the scale.

To find the logarithm of a number, set the left hand index of C to the number on D and read the mantissa on L against the index line in the slot.

Example 89. Determine the logarithms of 8·2, 4·15 and 3·2.

Set the left hand index of C to 8·2 on D, and read 0·914, the required logarithm, on L against the L index line.

Set the left hand index of C to 4·15 on D, and read 0·618 on L.

Set the left hand index of C to 3·2 on D, and read 0·505 on L.

Example 90. Determine the logarithms of 82, 415 and 3200.

The slide is put into the same positions as in example 89, and the mantissae are read on L as before. The characteristics 1, 2, and 3 are found as previously explained, and the logarithms are 1·914, 2·618 and 3·505.

Example 91. Find log 13200.

Setting the left hand index of C to 13200 ⟨1·32⟩ on D, the mantissa, 0·121, is read on L against the L index; the characteristic is 4, so log 13200 = 4·121.

Logarithms are particularly useful in solving problems involving roots and powers; by their aid any root or power, fractional or otherwise, may be quickly obtained.

Example 92. Raise 2·25 to the sixth power.

Setting the left hand index of C to 2·25 on D, the mantissa on the L scale is seen to be 0·352; as the characteristic is 0, the right hand index is set to 0·352 ⟨3·52⟩ on D and against 6 on C 2·11 is found on D. This is log $2{\cdot}25^6$, and to take the antilog, set 0·11 on L to the L index line and read 129, the significant figures in the power; since the characteristic is 2, the answer is 129.

Example 93. $24^2 = 576$.

Set the left hand index of C to 24 on D and read 0·38 against the L index line. Since the characteristic is 1, log 24 = 1·38. Multiplying this by 2, we get 2·76; set 0·76 to the L index line and the significant figures, 576, are read on D; since the characteristic is 2, the answer will be 576.

EXAMPLE 94. $19^9 = 3{\cdot}24 \times 10^{11}$.

Set the left hand index of C to 19 on D and read 0·279 on L, the characteristic is 1, hence log 19 = 1·279. Set the right hand index of C to 1·279 on D and read 11·51 on D against 9 on C, this is log 19^9. Bring 0·51 on L to the L index line and read 324 on D as the significant figures in the answer, which is thus 324,000,000,000. A convenient way of doing this latter portion would be to find the number on D corresponding to 0·51 on L, this is 3·24, so log 3·24 = 0·51. This is then multiplied by 10^{11}, giving $3{\cdot}24 \times 10^{11}$ as the answer.

Roots are found by dividing the logarithm of the number by the exponent of the root.

EXAMPLE 95. $\sqrt[7]{283} = 2{\cdot}24$.

Following the method above suggested for the last example, this may be written $\sqrt[7]{2{\cdot}83 \times 10^2}$, the logarithm of 2·83 is found on L and, since the second factor is 10^2, the characteristic of the whole expression will be 2.

Log 2·83 = 0.452.
Log 283 = 2·452.

Dividing this by 7 on the lower scales we find the quotient, 0·35, on D. Setting this on L to the L index we read the answer, 2·24, on D.

EXAMPLE 96. $\sqrt[5]{1280} = 4{\cdot}18$.

Log 1·28 = 0·108 as read on L.
Log 1280 = 3·108.

Dividing by 5 we get 0·622; the number corresponding to this logarithm is found on D and is 4·18.

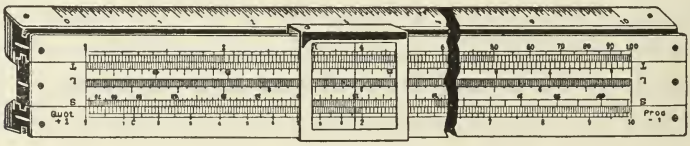


Fig. 4

If the slide be removed from the rule and replaced with the L scale to the front, Fig. 4, so that 1 on L is to 10 on D and 0 is against 1, numbers on D will be in line with their logarithms on L. In this position the figures on L are inverted. Unless many logarithms are required this method is not to be recommended, as the slide cannot be used for multiplication or division, though it is generally found easier to read the logarithms.

For practice the answers to combined multiplication and division problems may be checked by logarithms.

EXAMPLE 97. $\dfrac{246 \times 1{\cdot}4 \times 81 \times 23}{0{\cdot}008 \times 63 \times 146 \times 28}$

Log 246	= 2·391	Log 0·008	= $\bar{3}$·903
Log 1·4	= 0·146	Log 63	= 1·799
Log 81	= 1·908	Log 146	= 2·164
Log 23	= 1·362	Log 28	= 1·447
Adding	5·807	Adding	3·313

	5·807
	3·313
Subtracting	2·494

Setting 0·494 on L to the L index, the answer, 312, is read on D This is example 34 on page 17, where the answer, as found on the slide rule, is given as 311.

It will be seen that the logarithmic method of doing multiplication and division takes considerably longer than the direct slide rule method, and thus the latter is to be preferred.

As a further demonstration of the uses of the scale of logarithms on the slide rule, the following example on compound interest will be fully explained.

EXAMPLE 98. Determine the amount of £ 400 for 7 years at 5 per cent per annum compound interest.

At the end of 1 year £ 1 at 5 per cent is £ 1·05. Set the left hand index of C to 1·05 on D, and the logarithm, 0·0212, is read on L. Bring the right hand index of C to 0·0212 on D and read 0·148 on D against 7 years on C. Put 0·148 on L to the L index, and the answer £ 563 is found on D under £ 400 on C.

With this last setting any principal may be taken at 5 per cent for 7 years by moving the cursor over the required sum on C, and reading the answer on D.

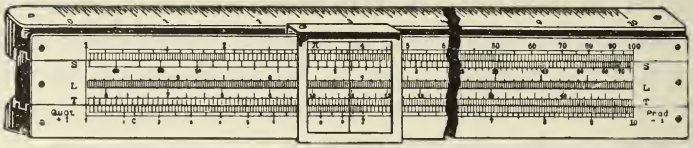


Fig. 5

TRIGONOMETRICAL SCALES.

On the back of the slide, in addition to the scale of logarithms L, will be found two other scales marked S and T. These scales are used for reading sines, tangents, and other trigonometrical functions. Inspection of these scales will show that the graduations represent degrees and minutes of angles, the corresponding sines and tangents are given on the front of the slide and on scale D, with the slide in its normal position; but if these scales be brought to the front of the rule by reversing the slide, so that S is against A, and T is against D, Fig. 5, then A gives sines and D gives tangents.

The Scale of Sines.

The first graduation after the left hand index of S is 35′, then 40′ and increasing by 5 minutes to 10°; from 10° to 20° the division lines are 10 minutes apart; from 20° to 40° the spacing is 30 minutes; above 40° the increase is by one degree. On the sine scale S it is impossible to read angles above 80°, and when accurate results are required for angles greater than 60° a method, which will be explained later, should be used.

Reversing the slide, as above explained, the angles on S lie adjacent to their sines on A, Fig. 5. Since sine 5° 43′ is 0·1 the sines of angles less than 5° 43′ have a cypher directly following the decimal point, and are read on the left hand A scale. For angles above 5° 43′ the sines are found on the right hand A scale, and the first significant figure follows the decimal point. This will be understood from the following readings.

Sin $1^0\ 45'$	$= 0{\cdot}0305$	Read on the left hand A scale.
Sin $2^0\ 30'$	$= 0{\cdot}0436$	
Sin $3^0\ 15'$	$= 0{\cdot}0567$	
Sin 4^0	$= 0{\cdot}0698$	
Sin 5^0	$= 0{\cdot}087$	
Sin $6^0\ 20'$	$= 0{\cdot}11$	Read on the right hand A scale.
Sin $8^0\ 30'$	$= 0{\cdot}1478$	
Sin 20^0	$= 0{\cdot}342$	
Sin 30^0	$= 0{\cdot}5$	
Sin 55^0	$= 0{\cdot}819$	

When a number has to be multiplied or divided by the sine of an angle, the number is taken on A, and the angle on S is used instead of the sine. Since scale S is graduated to correspond with its sines on A when the A and S indices are in line, the angles on S, considered as lengths from the left hand index, are equal to the sines.

Example 99. $28 \sin 56^0 = 23{\cdot}2$.

Set the right hand index of S to 28 on A and read the answer, 23·2, on A over 56^0 on S.

Example 100. $\dfrac{228}{\sin\ 16^0\ 30'} = 803$.

Setting $16^0\ 30'$ on S to 228 on A we read the quotient, 803, on A over the right hand index of S.

No difficulty will be experienced in locating the position of the decimal point.

It is not always convenient to reverse the slide in order to find the sine of an angle. Looking at the back of the rule, with the slide in its normal position, Fig. 3, a small black line is seen on the edge of the slot at each end against the S scale. These lines will be called the S index lines, and by bringing the angle on S to either, the sine is found on B under the right or left hand index of A. If we use the right hand S index line we read the sine under the right hand A index, and vice versa.

Example 101. Find the sine of 38^0.

Set 38^0 on S to the S index line and read 0·616 on B under the A index

EXAMPLE 102. Sin $4^0\ 15' = 0{\cdot}074$.

This, being read on the left hand B scale, requires a cypher after the decimal point.

Multiplication of sines may be simply effected with the slide in its normal position by setting the angle on S to the S index line and reading the answer on B against the other factor on A.

EXAMPLE 103. $4 \sin 38^0 = 2{\cdot}46$.

Set 38^0 to the S index and against 4 on A read 2·46 on B.

Division of a number by the sine of an angle is carried out by putting the angle on S to the S index line, when the answer will be found on A above the number on B.

EXAMPLE 104. $\dfrac{28}{\sin 30^0} = 56$.

Set 30^0 on S to the index line and against 28 on B read 56 on A.

For angles larger than 60^0 the sine may be found by the following formula:

$$\text{Sin}\ \Theta = 1 - 2 \sin^2 \left(\frac{90^0 - \Theta}{2}\right)$$

EXAMPLE 105. Find the sine of 80^0.

$$\text{Sin}\ 80^0 = 1 - 2 \sin^2 \left(\frac{90^0 - 80^0}{2}\right)$$
$$= 1 - 2 \sin^2 5^0.$$

Set 5^0 to the S index line and read 0·087 on B against the index of A. To find $\sin^2 5^0$ bring the cursor over $\sin 5^0$ ⟨0·087⟩ on A and read the square 0·0076 on B.

Sin 80^0 then is $1 - 2 \times 0{\cdot}0076$
$= 0{\cdot}9848$.

Doing the above example with the slide reversed, $\sin 5^0$ is found on A, and the left hand index of S is brought to it, when $\sin^2 5^0$ will be found on A above 5^0 on S.

Cosines of Angles are found from the formula

$$\text{Cos}\ \Theta = \sin \langle 90^0 - \Theta \rangle.$$

Thus to find the cosine of an angle, the number of degrees in the angle are subtracted from 90^0, and the sine of the remaining angle is found as above described; this is the required cosine.

EXAMPLE 106. Find the cosine of 68^0.

$$\text{Cos}\ 68^0 = \sin \langle 90^0 - 68^0 \rangle$$
$$= \sin 22^0$$
$$= 0{\cdot}375.$$

Since the cosecant equals the reciprocal of the sine, the cosecant is found on A against the index of B when the angle is set to the S index. One setting gives both sines and cosecants.

In the same way the secant is read on A against the index of B when the cosine is set on B against the index of A.

Both cosecants and secants are greater than unity.

On Rules No. 317, 375, 386, 387, 397 and 385 N, the S scale is used with the lower scales.

The Scale of Tangents.

By reversing the slide so that the T scale lies alongside the D scale, Fig. 5, a table of tangents of angles from $5^0\ 43'$ to 45^0 is formed; the angles being on T and the tangents on D. Between these limits the tangents are between 0·1 and 1. For angles less than $5^0\ 43'$, the sine may be taken instead of the tangent, the error involved with such small angles being a negligible quantity. Angles larger than 45^0 have tangents above unity, and cannot be directly read off. In this case the following formula must be used:

$$\text{Tan}\ \ominus = \frac{1}{\tan \langle 90^0 - \ominus \rangle}.$$

This is easily carried out by bringing the complement of the required angle on T to the left hand index of D, and reading the tangent on D against the index of T.

Example 107. Find tan $72^0\ 30'$.

$$\text{Tan}\ 72^0\ 30' = \frac{1}{\tan \langle 90^0 - 72^0\ 30' \rangle}$$

$$= \frac{1}{\tan \langle 17^0\ 30' \rangle}$$

Setting $17^0\ 30'$ on T to the index of D, we read the tangent, 3·17, on D against the index of T.

Example 108. Tan $65^0 = 2{\cdot}14$.

$$\text{Tan}\ 65^0 = \frac{1}{\tan \langle 90^0 - 65^0 \rangle}$$

$$= \frac{1}{\tan 25^0}.$$

Bringing 25^0 on T to 1 on D, we read the tangent, 2·14, on D against the right hand index of T.

When the angle exceeds $\langle 90^0 - 5^0\ 43' \rangle = 84^0\ 17'$ the tangent exceeds 10, and so cannot be read on scale D. In this case the

complementary angle is less than $5^0\ 43'$, consequently the sine is substituted for the tangent of the complementary angle, the angle is subtracted from a right angle, and the remaining angle on S is set against the index of A and the tangent is read on A against the S index. It must be clearly understood that scales S and T, although marked in degrees and minutes, are graduated with lengths proportional to the logarithms of sines and tangents respectively, hence, by setting the complementary angle on S or T against 1 on A or D, the reciprocal of the sine or tangent of the complementary angle is found.

Example 109. Find tan 86^0.

$$\text{Tan } 86^0 = \frac{1}{\tan \langle 90^0 - 86^0 \rangle}$$

$$= \frac{1}{\tan 4^0}$$

And may be taken as $\dfrac{1}{\sin 4^0}$.

Set 4^0 on S to 1 on A and read 14·3 — the required tangent — on A against the index of S.

With the slide in its normal position, Fig. 3, the tangent may be found by setting the angle on T to the T index line in the slot at the left hand end of the rule, and reading the tangent on C against the index of D.

Example 110. Find tan 15^0.

Set 15^0 on T to the T index line and read the tangent, 0·268, on C over 1 on D. The reading, 3·73, on D against the index of C is tan $\langle 90^0 - 15^0 \rangle = \tan 75^0$.

Example 111. Find tan 38^0.

Set 38^0 on T to the T index line and read the tangent, 0·781, on C over 1 on D. With this setting tan $\langle 90^0 - 38^0 \rangle = \tan 52^0$ is read on D under the index of C, and is 1·28.

From these examples it will be seen that each setting with the slide normal gives tan $\ominus$ and tan $\langle 90^0 - \ominus \rangle$, and since tan $\langle 90^0 - \ominus \rangle$ = cotangent $\ominus$, it follows that each setting gives tangents and cotangents. For angles less than 45^0, the tangents are read on C

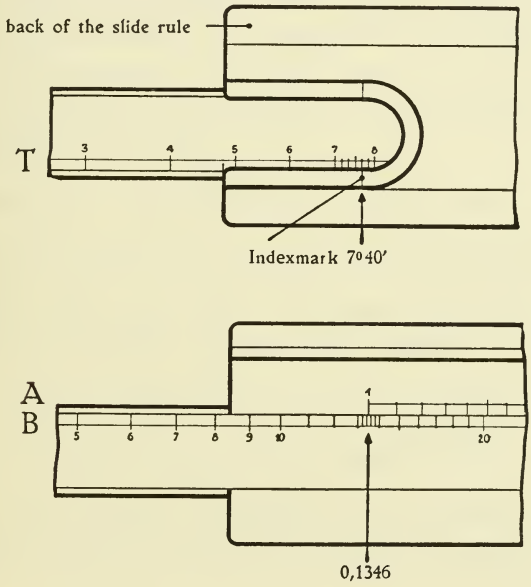
and are under 1, while the cotangents are read on D and are greater than 1; above 45^0 cotangents are on C and tangents on D. This will be obvious when it is remembered that $\tan \ominus = \frac{1}{\text{Cot} \ominus}$.

Multiplication and division with tangents are done in the same way as with sines.

The method of finding the angle from the sine or tangent, etc. will be quite clear from the foregoing.

On Rules No. 380 N, 382 and 388 N, the T scale is used with the upper scales. In these cases the procedure is somewhat different from the above and the result is read on scale B. This will be understood from the following example.

EXAMPLE: Tan $7^0 40' = 0 \cdot 1346$.



Mark ϱ' and ϱ".

The marks ϱ' and ϱ" are provided for reading the functions of very small angles.

Both are found on scale C, ϱ' being between 3·4 and 3·5, and ϱ" between 2 and 2·1.

ϱ' is used when the angle is in minutes, ϱ" when it is in seconds.

In the case of small angles, the trigonometrical functions, sine and tangent, are almost identical with the arc.

EXAMPLE: Sin 17′ ~ tan 17′ arc 17′ = 0·00495.

Set the mark ϱ′ over 1·7 on D and read the function on D under 10 on C.

EXAMPLE: Sin 42″ ~ tan 42″ ~ arc 42″ = 0·0002085.

Set the mark ϱ″ over 4·2 on D and read the function on scale D under 1 on C.

With the slide rules that are fitted with the mark for centesimal measure ⟨100ᵍ to the quadrant⟩, the same graduation ⟨between 6·3 and 6·4 on C⟩ is used for centesimal minutes and seconds.

EXAMPLE: Sin 0·17ᵍ ~ tan 0·17ᵍ ~ arc. 0·17ᵍ = 0·00267.

Sin 0·0042ᵍ ~ tan 0·0042ᵍ ~ arc 0·0042ᵍ = 0·000066.

APPLICATIONS.

Time will be saved and greater accuracy attained if the movements of the slide be reduced as far as possible. On this account it is profitable to consider a problem before calculating to see whether a rearrangement of the terms would not simplify the settings.

When squares and cubes occur in multiplication and division the upper scales are generally used and the square worked in by means of scales C and B. For instance, in calculating the areas of circles from the diameter, $a = \frac{\pi}{4} d^2$, where a an d are area and diameter respectively. Set 4 on B to π on A and over the diameter on C read the area on A.

The volume of a sphere is given by $\frac{\pi}{6} d^3$. Set 6 on B to π on A and over the diameter on scale C will be found $\frac{\pi}{6} d^2$ on A; putting the cursor over this and bringing the index of B to the cursor line, we read the volume over d on B.

On the lower scales will be found two gauge points c and c_1 the former equals $\sqrt{\frac{4}{\pi}}$ and the latter $\sqrt{\frac{40}{\pi}}$. These are used in finding the volumes of cylinders. The volume of a cylinder, $V = \frac{\pi}{4} d^2 L$, where d and L are diameter and length. By setting c on scale C to the diameter on D we obtain $\frac{d}{\sqrt{\frac{4}{\pi}}}$ which, on scale A above is $\frac{d^2}{\frac{4}{\pi}} = \frac{\pi}{4} d^2$, and on setting the cursor over L on B, the volume is read under the cursor line on A. When the length cannot be found on B due to the projection of the slide to the right, the gauge point c is replaced by c_1. Volumes found by

means of c_1 must be multiplied by 10. This will be apparent from the relative sizes of c and c_1.

EXAMPLE 112. Find the volume of a cylinder 8 inches in diameter and 30 inches long.

Set c_1 on scale C to the diameter 8″ on D and against 30″ ⟨3⟩ on B read 1500 ⟨15⟩ cubic inches on A. By using c_1 instead of c and 3 in place of 30, the reading, 15, must be multiplied by 100. With the above setting, the cross sectional area of the cylinder, 50 sq. inches, was given on A against the index of B.

To find the area of a circle using the gauge points: set c on scale C to the diameter on D and read the area on A against the left hand index of B.

The same result is achieved by means of the three line cursor. The distance between the middle and right hand lines is the same as from 1 to c, consequently, if the right hand line be set over the diameter on D, the area will be found on A under the centre cursor line.

Since the volume of a sphere of diameter d is $\frac{2}{3}$ of the volume of a cylinder, whose length and diameter are each equal to d, the volume of a sphere may be found by means of c or c_1; the volume of the cylinder is calculated and $\frac{2}{3}$ of it will be the volume of the sphere.

EXAMPLE 113. Determine the volume of a sphere 8 inches in diameter.

Set c_1, on scale C to 8 on D and put the cursor over 8 on B, bring 3 on B under the cursor line and read the volume, 268 cubic inches, on A over 2 on B.

EXAMPLE 114. Find the capacity in gallons of a cylindrical tank which is 8 feet in diameter and 11 feet high.

Set c_1 on scale C to 8 on D and against 11 on B read 554 cubic feet on A. Leaving the cursor over 554 on A, we find on the back of the rule that 17 cubic feet equal 106 gallons. Setting 106 ⟨1·06⟩ on B to 17 ⟨1·7⟩ on A we read 3450 gallons on B under the cursor.

By means of the gauge point M, which equals $\frac{100}{\pi}$, on the upper scales, it is possible to find the area of curved surface and the circumference of a cylinder from the diameter and length with one setting. The curved surface of a cylinder is given by $\pi\, d\, L$, where d and L are diameter and length. By changing $\pi\, d\, L$ to $\frac{d}{\frac{100}{\pi}}\, L$, a multiplication follows a division, and one setting gives both answers.

Example 115. Determine the area of curved surface and the circumference of a cylinder 10 inches in diameter and 15 inches long.

Set M on B to 10 on A and, against 15 on B, read 471 square inches, the required area, on A. Against the index of B read the circumference, 31·4 inches, on A.

Areas of similar figures are to each other as the squares of their respective linear dimensions.

Example 116. The area of an equilateral triangle of side 1 is 0·433 and its height is 0·866. What is the area of a similar triangle whose height is 1?

$$0{\cdot}866^2 : 1^2 :: 0{\cdot}433 : x$$

In this example, the first term is taken on C, the second on D, the third on B and the fourth on A. Set 0·866 on C to 1 ⟨10⟩ on D and against 0·433 on B read x = 0·577 on A.

To determine the pitch of a gear wheel: set the number of teeth on B to π on A, and against any pitch diameter on B read the corresponding pitch on A.

To find the diameter of a gear wheel: bring π on B to the number of teeth on A, and against any pitch on B read te diameter on A.

To find the number of teeth in a pinion to have a given velocity: set the velocity of pinion on B to the number of teeth of driver on A, and, against any velocity of driver on B, read the required number of teeth on the pinion on scale A.

Example 117. The driver has 60 teeth and makes 21 revolutions per minute, find the number of teeth on the pinion, which revolves at 35 per minute.

Set 35 R.P.M. on B to 60 teeth on A and against 21 R.P.M on B read 36 teeth on A.

The equation $T = \pi \sqrt{\frac{L}{g}}$ would be conveniently solved as follows:

Set g on B to L on A, move the cursor over 22 on C, bring 7 on C under the cursor and read the answer on D against the index of C. By this means we obtain the square root of $\frac{L}{g}$ at once on D.

EXAMPLE 118. $\frac{\sqrt{3} \times 550}{\sqrt{2} \times 2} = 337.$

Set 2 on B to 3 on A and move the cursor over 550 on C, bring 2 on C under the cursor and read 337 on D, against the left hand index of C.

EXAMPLE 119. For a three phase alternating current motor the power $P = \sqrt{3}\ E\ I \cos \theta$. Find P when $E = 500$ volts, $I = 15$ amperes, and $\theta = 37^0$.

Commencing with the last term, $\cos 37^0 = \sin \langle 90^0 - 37^0 \rangle = \sin 53^0$. Set 53^0 on S to the S index line and read 0·8 on B against the index of A. Bring the right hand index of C to 0·8 on D, set the cursor over 3 on B, put the left hand index B to the cursor, move the cursor over 500 on C, bring the right hand index of C under the cursor and read 10·4 kilowatts on D, against 15 on C.

The indicated horsepower of a steam-engine is given by the formula $\text{I. H. P.} = \frac{PLAN}{33000}$, in which P = mean effective pressure, in pounds per square inch.

L = length of stroke in feet.
A = area of piston, in square inches.
N = number of working strokes, per minute.

When the diameter of piston and the revolutions per minute are given for a double-acting engine. the formula for slide rule working may be simplified as follows:

$\text{I.H.P.} = \left(\frac{d}{c}\right)^2 \times \frac{L}{0{\cdot}5} \times \frac{P\,N}{33000}$. In the first factor, *d* is diameter of piston, and *c* is the gauge point on scale C. The divisor in the second factor is the reciprocal of 2. By dividing by 0·5 instead of multiplying by 2, we save an extra setting of the slide.

In calculating this formula the gauge point c or c_1 on scale C is placed against d on scale D and the cursor is moved over L on B, 0·5 on B is brought under the cursor, the cursor is moved over P on B, 33000 is placed under the cursor; the answer will now be found on scale A over N on B.

Example 120. Determine the indicated horsepower of a steam-engine having a piston of 20 inches diameter and a stroke of 2 feet; the mean effective pressure is 35 pounds per square inch, and the speed is 120 revolutions per minute.

$$\text{I.H.P.} = \left(\frac{20}{c}\right)^2 \times \frac{2 \times 35 \times 120}{0{\cdot}5 \times 33000} = 160 \text{ H. P.}$$

Given horsepower, mean effective pressure, and piston speed, to find the diameter of the cylinder, set the mean effective pressure on B to the indicated horsepower on A, move the cursor over the index of B, bring the piston speed on B under the cursor, move the cursor over 205 on C, and read the diameter of cylinder on D under the cursor.

Example 121. An analysis of a sample of coal gave the following:

2·08 % moisture,
7·27 % volatile combustible matter,
74·32 % fixed carbon,
16·33 % ash.

Find the weight of each per ton of coal.

Set 100% ⟨1⟩ on scale C to 2,240 lb. on D and read 46·5 lb. on D under 2·08% on C. Again, read 365·7 lb. on D under 16·33% on C. Now, since the other two readings are too much to the right, re-set the slide by bringing 100% ⟨now 10⟩ on C to 2,240 on D and read 162·8 lb. and 1,665 lb. on D under 7·27% and 74·32% respectively on C.

Therefore, the required weigths are:

46·5 lb. moisture,
365·7 lb. ash,
162·8 lb. volatile matter,
1665·0 lb. fixed carbon.
2240·0

Calculation of Timber.

The number of feet run of any scantling to make a Standard equals $\frac{144 \times 165}{\text{Sectional area}}$. Set the sectional area in square inches on

scale B to 144 on scale A and read the length on scale A against 165 on B.

Example 122. Determine the feet run in a scantling 10" × 2¾" to equal a Standard.

Here the sectional area is 27·5 sq. in. Put the cursor over 144 ⟨1·44⟩ on scale A and set 27·5 on B under the cursor line, now move the cursor over 165 ⟨16·5, since 1·65 is too much to the left⟩ on B and read 864 feet on scale A under the cursor. That this is not 86·4 or 8640 will be apparent from a practical consideration of the problem.

Having found the feet run in a Standard of timber 27·5 square inches sectional area, one setting of the slide will convert any length of it into Standards. For instance, to find what decimal of a Standard will 250 feet of the above be, set the cursor over 250 ⟨2·5⟩ on scale A, and bring 864 ⟨8·64⟩ on B under the cursor line and read the answer, 0·29, on A over 100 on B.

To calculate the cubical contents of square timber: set the cursor over the length of a side in inches on D, bring 12 ⟨1·2⟩ on C under the cursor and move the cursor over the length in feet on scale B. The contents in cubic feet will be found under the cursor on A.

Example 123. Find the cubic feet in a balk, the side of which is 9" and the length 24 feet.

Set the cursor over 9 on D and bring 12 ⟨1·2⟩ on C to it, move the cursor over 1 on C and bring 10 on C under the cursor ⟨this becomes necessary since the slide is too much to the right for the next factor⟩, move the cursor over 24 on scale B and read 13·5 cubic feet on A under the cursor.

When the sides are not the same, bring the index of B to the thickness in inches on A, move the cursor over the width in inches on B, bring 144 on B under the cursor and against the length in feet on B, read the cubic feet on A.

The side of the equivalent square of unequal sided timber in inches is found by bringing the index ⟨1 or 100⟩ of B to the length of one side on A and moving the cursor over the length of the other side on B, when the length of the side of the square will be found on D under the cursor.

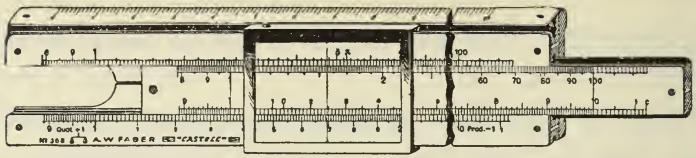
Example 124. What is the length of a side of the equivalent square of a piece of timber 18" × 5"?

Set the left hand index ⟨1⟩ of B to 18 on A and under 5 on B read 9·5 inches on D.

Example 125. Find the cost of 7 feet of timber 6″ × 9″ at 18 pence per cubic foot.

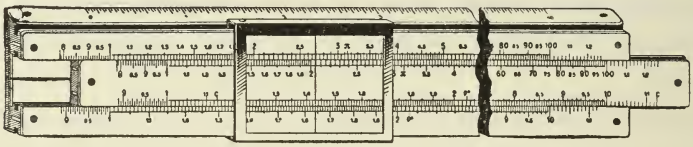
Set the left hand index of B to 7 on A, move the cursor over 6 on B, bring the right hand index ⟨100⟩ of B under the cursor line, put the cursor over 9 on B, bring 144 ⟨1·44⟩ on B under the cursor and move the cursor over 18 on B. The answer 47·25 pence will be found on A under the cursor. The cost therefore is 3/11¼ d.

As almost all technical calculations consist of multiplication, division, involution, evolution, or combinations of these, it will be apparent that no separate instructions will be required to enable any practical calculations to be worked on the slide rule once the general principles of slide rule operation are understood.



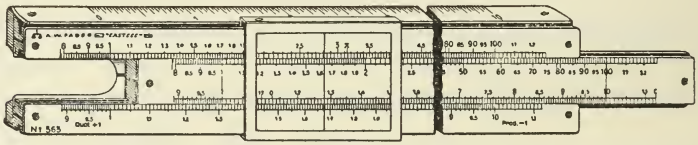
The A.W. FABER "CASTELL" Slide Rule No. 360.

This slide rule, which is made in five sizes, 4", 5", 6", 10" and 20", has all the scales described in the general instructions, and those instructions are a complete explanation of its working.



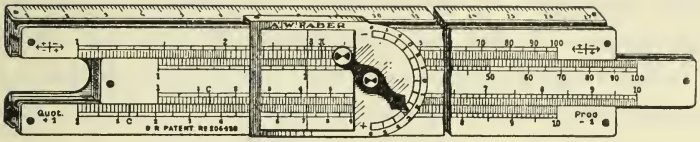
The A.W. FABER "CASTELL" Slide Rule No. 361.

This slide rule has been made for the use of students and is the same as No. 363 below, with the exception of the trigonometrical scales and scale of logarithms, which have been omitted. The general instructions relating to scales A, B, C and D are complete for this slide rule.



The A.W. FABER "CASTELL" Slide Rule No. 363.

This slide rule differs from No. 360 only in the extra numerals 1·1, 1·2, 1·3 etc. and 11, 12, 13, etc. which have been engraved on the four front scales against the graduations 1·1 to 1·9, and on scales A and B from 11 to 19. The general instructions are complete for this slide rule.



The A.W. FABER "CASTELL" Slide Rule No. 367. With digit registering cursor.

The rules for fixing the position of the decimal point in multiplication and division, when using the lower scales, have been given in the general instructions, and those who use these rules to determine the number of figures in their answers will have noticed that it is a matter of some difficulty to remember with certainty the various movements of the slide to the right. It has already been suggested on this account that a mark be made each time a quotient or product is obtained with the slide to the right, and that the marks for products be cancelled with those for quotients, and if any remain that they be added to, or subtracted from the number of digits in the problem, as the case may be. The digit registering cursor froms a convenient means of recording these movements of the slide to the right.

The digit registering cursor consist of an ordinary cursor having a semicircular projection as an extension of its top. Around the edge of this projection twelve division marks have been stamped, numbered 0 to + 6 downwards and 0 to — 6 upwards. A pointer has been arranged to move over these divisions and can be set to any one of them by a slight pus with the finger tip.

A few examples will explain the uses of this device for lower scale working.

EXAMPLE 1. $\dfrac{3 \times 28 \times 142 \times 16}{175 \times 156 \times 23 \times 14} = 0{\cdot}0216.$

We may commence in this case by setting the pointer to the difference between the number of figures in the denominator and numerator terms, which here is 8 — 10 = — 2. Set the pointer to — 2 and bring 175 on C to 3 on D, the slide project to the right in division, so the pointer is brought down to — 1. Move the

cursor over 28 on C, the pointer must be moved back to — 2, since the slide is to the right in multiplication. Bring 156 under the cursor line and the pointer to — 1. Move the cursor to 142 on C and the pointer to — 2. Bring 23 on C under the cursor and put the pointer to — 1. Put the cursor over 16 on C and the pointer to — 2. Set 14 on C under the cursor and put the pointer to — 1. The reading against the left hand index of C is 0·0216, since the pointer shows that — 1 figure is required.

EXAMPLE 2. $\dfrac{1}{12 \times 53 \times 1{\cdot}28} = 0{\cdot}00123.$

Here the number of figures is $1 - 5 = -4$. Set the pointer to — 4 and put 12 on C to 1 on D, move the cursor over 10 on C, bring 53 on C under the cursor and move the pointer to — 3, set the cursor over 1 on C, bring 1·28 under the cursor and push the pointer to — 2. The reading against the left hand index of C is thus 0·00123.

EXAMPLE 3. $\dfrac{76 \times 17 \times 320}{13 \times 29} = 1097.$

In this example the digits are $7 - 4 = 3$ to commence with. Set the pointer to + 3, bring 13 on C to 76 on D and move the pointer to + 4, move the cursor over 17 on C and put the pointer back to + 3, bring 29 on C under the cursor and the pointer to + 4, change ends of the slide, and put the cursor over 320 on C, when the answer, 1097, will be read on D under the cursor.

It has already been explained in the previous pages how, by suitably shifting the decimal points in multiplication and division, the number of figures in answers obtained on the upper scales may be determined without difficulty.

The sign $\leftarrow \begin{smallmatrix} + \\ - \end{smallmatrix} \Big| \begin{smallmatrix} - \\ + \end{smallmatrix} \rightarrow$ placed at each end of the A scale may be looked upon as a fraction, the arrows showing the directions in which the decimal points may be shifted, both in the numerator and denominator, and the + and — signs showing the directions in which the digit pointer must be set to compensate in the answer for these movements. If we shift the decimal point to the left in the numerator, the value of the whole problem is decreased and the answer must be increased accordingly; this is shown by the sign + above the arrow. If the decimal point is moved to the left in the denominator the whole value is increased and the answer must be correspondingly decreased; the sign — under the arrow to the

left acts as a reminder. The other two signs — an + on the right show that when the decimal point is moved to the right in the numerator the answer must be decreased, and when it is moved to the right in the denominator the answer must be increased. This little sign should be carefully considered, as it sets out, in a very small space, complete instructions for shifting the decimal points.

EXAMPLE 4. $820 \times 1220 = 1{,}000{,}000.$

If the first factor be changed to 8·2, the digit pointer should be moved to + 2, and the second factor may be taken as 1·22 with the digit pointer moved + 3 places more to + 5. Set the left hand index of B to 8·2 on A and against 1·22 on B read 10 on A. The digit pointer shows that the decimal point must be moved 5 places to the right, hence the answer is 1,000,000.

EXAMPLE 5. $\frac{3900}{142} = 27{\cdot}5.$

This may be altered to 3·9 with the digit pointer on + 3, divided by 1·42 with the digit pointer set — 2 places more, that is to + 1. Setting 1·42 on B to 3·9 on A we read 2,75 on A against the left hand index of B. Moving the decimal point one place to the right as shown by the digit pointer, we get the answer 27·5.

EXAMPLE 6. $\frac{3 \times 28 \times 142 \times 16}{175 \times 156 \times 23 \times 14} = 0{\cdot}0216.$

Taking the numerator terms first, we get 3 with the pointer on 0, 2·8 with the pointer on + 1, 1·42 with the pointer on + 3, 1·6 with the pointer on + 4. The denominator terms change to 1·75 with the pointer on + 2, 1·56 with the pointer on 0, 2·3 with the pointer on — 1, 1·4 with the pointer on — 2. Leaving the pointer on — 2, we proceed to calculate the above example, each term of which is a whole number under 10, and so the left hand B scale is used throughout. The working on the rule gives 2·16 and the pointer shows — 2, so the answer will be 0·0216. This is the same as Example 1, which was worked on the lower scales.

We have, so far, been able to multiply and divide using the left hand index ⟨1⟩ of B only, but each time the right hand index ⟨100⟩ is used for multiplication, the answer has to be multiplied by 100. When the right hand index of B has to be used in division, the quotient must be divided by 100.

EXAMPLE 7. $\frac{746 \times 325}{950} = 255.$

This becomes 7·46, with the pointer on + 2, mutiplied by 3·25, with te pointer on + 4. divided by 9·5, with the pointer back on + 2. Setting 9·5 on B to 7·46 on A, we read 2·55 on A above 3·25 on B. Here the right hand index of A was used once each in multiplication and division, thereby cancelling, and, since the digit pointer shows + 2, the answer is 255.

EXAMPLE 8. $\frac{346 \times 22}{17 \times 75} = 5{\cdot}98.$

We get 3·46, with the pointer on + 2, multiplied by 2·2, with the pointer on + 3, for the numerator terms, and 1·7, with te pointer on + 2. multiplied by 7·5, with the pointer on + 1, for the denominator. Leaving the pointer on + 1, we set 1·7 on B to 3·46 on A and move the cursor over 2·2 on B. Now bring 7·5 under the cursor and read 59·8 on A against the right hand index of B. The right hand index of B was used in division and the pointer shows + 1. so the answer is $\frac{59{\cdot}8 \times 10}{100} = 5{\cdot}98.$

EXAMPLE 9. $\frac{0{\cdot}62}{0{\cdot}008} = 77{\cdot}5.$

Moving the decimal points, we get 6·2 with the pointer on — 1, divided by 8 with the pointer on + 2. The reading on A over the right hand index of B is 77·5. Therefore, the answer is $\frac{77{\cdot}5 \times 100}{100} = 77{\cdot}5$, since the right B index was used and the pointer showed + 2.

Squares and Square Roots.

In squaring a number using the digit registering cursor, we move the decimal point sufficient places to bring the number within the scope of the D scale, that is, between 1 and 10. The pointer is moved to twice the number of places that the decimal point was shifted. The square of the modified number will be found on scale A, and the point is then moved in accordance with the record shown on the digit registering scale.

EXAMPLE 10. Square 186.

Move de decimal point two places to the left and the pointer to + 4. Setting the cursor over 1·86 on D, the square is seen to

be 3·46, and, moving the decimal point four places to the right, we get the answer, 34,600.

Example 11. Square 0·0038.

Moving the decimal point three places to the right, we get 3·8 with the digit pointer on — 6. The square of 3·8 is found to be 14·4 and, moving the decimal point 6 places to the left, the answer is 0·0000144.

The digit registering cursor may be used to advantage in extracting square roots, as, by moving the decimal point an even number of places, any number may be reduced to such dimensions that it can be taken as marked on scale A. The digit pointer is set to half the number of places that the decimal point was moved, and the root of the altered number is found on D, the pointer showing the number of figures to be added.

Example 12. $\sqrt{784} = 28$.

Set the pointer to + 1 and find the root of 7·84, which is seen to be 2·8. The required root is 28.

Example 13. $\sqrt{0{\cdot}00034} = 0{\cdot}0184$.

Se the pointer to — 2 and take the root of 3·4, which is 1·844; move the point two places to the left, as shown by the pointer, and the answer is 0·01844.

Cubes and Cube Roots.

The methods of applying this device to cubes and cube roots should be clear from the foregoing and, therefore, requires only a short explanation.

In cubing a number, the decimal point is moved until the value of the number is between 1 and 10, and thus may be found on scale D. To compensate in the answer for this movement, the pointer is moved three places for each place the decimal point was shifted in the number — positive for a whole number and negative for a decimal.

Example 14. Find the cube of 28.

Set the digit pointer to + 3, and the cube of 2·8 is found to be 21·9, moving the decimal point three places, as shown by the pointer, brings the answer to 21900.

Example 15. Cube 0·041.

Set the pointer to — 6 and find the cube of 4·1, which is 68·9, move the decimal point six places to the left, and the answer is 0·0000689.

In extracting cube roots, if the number contains more than three figures it may be reduced, by moving the decimal point three or six places, to such value that the root will be found on scale D. To compensate for such movement the pointer is set one division — positive for a whole number, negative for a decimal — for each three places that the decimal point was shifted, and when the root of the changed number has been read on D, the pointer shows how far the decimal point must be moved, and in what direction.

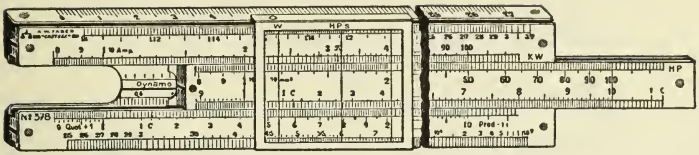
Example 16. Find the cube root of 18600.

Put the pointer + 1 and find the cube root of 18·6, which is 2·65. Moving the decimal point one place to the right, we get the answer 26·5.

Example 17. Extract the cube root of 0·0004.

Setting the pointer to — 2, we must move the decimal point six places to the right. The cube root of 400 is found to be 7·37, and moving the decimal point back two places, we get 0·0737 as the required root.

The other operations are carried out on this slide rule exactly as explained in the general instructions.



The A.W. FABER "CASTELL" Slide Rule No. 378. For Electrical and Mechanical Engineers.

Also applies to Slide Rule No. 398 ‹see Page 69›.

This slide rule is fitted with all the ordinary scales mentioned in the general section as well as three extra scales, which greatly extend its scope and enable complicated mathematical expressions to be solved with quickness and accuracy. Also two gauge points have been added which have special significance in electrical calculations. By means of the extra cursor lines watts are obtained from horsepower and vice versa, without using the slide. The left hand cursor line may be used in place of gauge point C, as already explained in the chapter on applications.

The Log-Log Scale.

The log-log scale has been arranged in two parts along the top and bottom edges of the front of the rule – above the A scale and below the D scale.

By multiplying the logarithm of a number by the index of a power, the logarithm of the number raised to that power is obtained. This may be done on the ordinary slide rule scales by finding the logarithm on the L scale, multiplying it by the index of the power required on scales C and D, then transferring the mantissa portion of the product to the L scale and finding the power on D.

Since involution and evolution are simplified by logarithms to multiplication and division, it will be clear that by taking the logarithm of the number's logarithm and adding it to the logarithm of the index of the power, the logarithm of the logarithm of the number raised to the required power will be obtained. The expression "logarithm of the logarithm" is generally abbreviated to log-log. Conversely, by subtracting the logarithm of the index from the log-log of the

number, the log-log of the root is found. Calling the number x, and the index of the power or root a, we get

$$\log x^a = a \log x,$$

$$\text{and } \log a + \log \langle \log x \rangle = \log \langle \log x^a \rangle.$$

$$\text{Also } \log \sqrt[a]{x} = \frac{\log x}{a},$$

$$\text{and } \log \langle \log x \rangle - \log a = \log \langle \log \sqrt[a]{x} \rangle.$$

The extra scale upon the face of this rule is graduated to give lengths proportional to the log-log of numbers from 1·1 to 100000, and as scale C is graduated with lengths proportional to the logarithms of numbers between 1 and 10, so the extra log-log scale used in combination with scale C enables all powers and roots to be found directly upon the face of the slide rule with one setting of the slide, provided the numbers and answers are between 1·1 and 100000, and, by means of a suitable multiplication or division by a multiple of 10, any number may be worked on this scale.

The log-log scale is divided into two parts — upper and lower. The upper log-log scale extends from 1·1 to 3·2, while the lower one commences at 2·5 and ends at 100000, the portion of the upper scale from 2·5 to 3·2 being repeated on the lower. The relative values of the graduations on the two portions of this scale are such, that each division on the upper log-log scale is the tenth root of the one exactly under it on the lower log-log scale. Thus, to find the tenth root of a number, set the cursor over it on the lower portion of the log-log scale and read the root on the upper; to find the tenth power of a number, take the number on the upper scale and read the power on the lower.

EXAMPLES 1. ⟨a⟩ $2{\cdot}1^{10} = 1668.$

⟨b⟩ $3^{10} = 59{,}000.$

⟨c⟩ $\sqrt[10]{25} = 1{\cdot}38.$

⟨d⟩ $\sqrt[10]{100} = 1{\cdot}585.$

⟨e⟩ $\sqrt[10]{1000} = 1{\cdot}995.$

Should the number not be within the limits of the log-log scale, it may be factorised and the factors worked separately.

EXAMPLES 2. ⟨a⟩ $25^{10} = 2{\cdot}5^{10} \times 10^{10} = 9540 \times 10^{10}.$

⟨b⟩ $0{\cdot}25^{10} = \dfrac{2{\cdot}5^{10}}{10^{10}} = 9540 \times 10^{-10}.$

⟨c⟩ $9^{10} = 3^{10} \times 3^{10} = 59000^2 = 34{\cdot}8 \times 10^8.$

$$\langle d\rangle\ \sqrt[10]{150{,}000} = \sqrt[10]{150} \times \sqrt[10]{1000} = 1{\cdot}65 \times 1{\cdot}995 = 3{\cdot}29.$$

$$\langle e\rangle\ \sqrt[10]{2} = \sqrt{\frac{20}{10}} = \frac{\sqrt[10]{20}}{\sqrt[10]{10}} = \frac{1{\cdot}35}{1{\cdot}259} = 1{\cdot}072.$$

Scale D in combination with the log-log scale gives a table of hyperbolic logarithms of numbers from 1·105 to 22,000. Numbers up to 2·718 are taken on the upper log-log scale, and log *e* is read on D and is less than 1. For numbers above 2·718, the lower log-log scale is used and log e, which is still found on scale D, is above 1. Log *e* 2·718 equals 1, and log *e* 22000 equals 10.

Under every number N on scale D will be found e^N on the lower log-log scale.

EXAMPLES 3. ⟨a⟩ $e^2 = 7{\cdot}39.$
⟨b⟩ $e^3 = 20{\cdot}1.$
⟨c⟩ $e^5 = 148.$
⟨d⟩ $e^{2{\cdot}64} = 14.$
⟨e⟩ $e^{7{\cdot}1} = 1200.$
⟨f⟩ $e^{10} = 22{,}000.$

The above values are found on the log-log scale under 2, 3, 5, 2·64, 7·1, and 10 on D.

Powers higher than the tenth may be obtained by factorising the index and raising *e* to the power shown by the first factor, the answer to this then being raised to the power as indicated by the second factor.

EXAMPLE 4. $e^{12} = \langle e^6\rangle^2 = \langle 402\rangle^2 = 162{,}000.$

Since each number on the upper log-log scale is the tenth root of the one directly beneath it on the lower log-log scale, and since any number N on D is over e^N on the lower log-log scale, it will be clear that a number N on D must be under $e^{\frac{N}{10}}$ on the upper log-log scale, so that above 2, 3, 4 etc., on D, the upper log-log scale gives $e^{0{\cdot}2}$, $e^{0{\cdot}3}$, $e^{0{\cdot}4}$, etc.

EXAMPLES 5. ⟨a⟩ $e^{0{\cdot}615} = 1{\cdot}85.$
⟨b⟩ $e^{0{\cdot}223} = 1{\cdot}25.$
⟨c⟩ $e^{0{\cdot}86} = 2{\cdot}36.$

Roots of e may be extracted by converting the exponent into a decimal and reading the root on the upper log-log scale over the decimal exponent on D.

EXAMPLES 6. ⟨a⟩ $\sqrt[2]{e} = e^{0\cdot5} = 1\cdot65.$

⟨b⟩ $\sqrt[5]{e} = e^{0\cdot2} = 1\cdot221.$

Roots of e may also be found by inverting the slide and using CR and the upper log-log scale, with the CR and A indices in line. The root is found above the exponent on CR.

EXAMPLES 7. ⟨a⟩ $\sqrt[2]{e} = 1\cdot65$, read above 2 on CR.

⟨b⟩ $\sqrt[1\cdot3]{e} = 2\cdot16$, read above 1·3 on CR.

Logarithms of numbers to any base may be found by bringing 1 on C to the base number on the log-log scale and reading, in line with the numbers on the log-log scale, the logarithm on C. For instance, by setting 1 on C to 10 on the log-log scale we read 1.301 on C in line with 20 on the log-log scale, 3 on C is in line with 1000, 4 is against 10,000, etc. We thus get a table of common logarithms, and may in the same manner arrange a table of logarithms to any base.

All roots and powers may be obtained by using the C scale and the log-log scale. Since the left hand index of D is exactly above e, or 2·718+, and we find the powers of e under the respective indices on D, it follows that by bringing the left hand index of C to any number x on the log-log scale, we find, under any number N on C, x^N on the log-log scale, provided, of course, that x^N is within the scope of the log-log scale. Should the power required be larger than 100000 the number must be factorised.

EXAMPLE 8. $7\cdot5^{2\cdot6} = 189.$

Bringing the left hand index of C to 7·5 on the log-log scale, we find the required power, 189, on the log-log scale under 2·6 on C.

EXAMPLES 9.

⟨a⟩ $2\cdot75^{1\cdot38} = 4\cdot04.$

⟨b⟩ $650^{1\cdot75} = 84{,}000.$

⟨c⟩ $28^9 = 2\cdot8^9 \times 10^9 = 10{,}600 \times 10^9.$

⟨d⟩ $1500^3 = 15^3 \times \langle 10^2 \rangle^3 = 15^3 \times 10^6 = 3380 \times 10^6.$

So long as the number is found on the lower log-log scale and the exponent is greater than unity, the left hand index of C must be used in finding powers, since the answer will be greater than the number. But when we find the number on the upper log-log scale and the required exponent on C is beyond the end of the rule with the slide projecting to the right, that is, the number is greater than 3·2, we must use the right hand index of C and read the power on the lower log-log-scale.

EXAMPLES 10. (a) $1{\cdot}25^{6} = 3{\cdot}82.$

(b) $1{\cdot}55^{8{\cdot}5} = 41{\cdot}5.$

An example such as $2{\cdot}1^{15}$ may be taken as $(2{\cdot}1^{10})^{1{\cdot}5}$. The term in the brackets is found on the lower log-log scale under 2·1, to which point the index of C is set for the next operation, and the answer, 68,000, is found on the lower log-log scale under 1·5 on C. Numbers less than 1·1 may be brought on to the log-log scale by moving the decimal point to the right and putting the number over one 10 for each place the decimal point was moved, and raising the numerator and denominator to the required power.

EXAMPLE 11. $0{\cdot}8^{1{\cdot}6} = \dfrac{8^{1{\cdot}6}}{10^{1{\cdot}6}} = \dfrac{27{\cdot}8}{40} = 0{\cdot}695.$

The extraction of roots should be comparatively easy from the foregoing, but care is required in deciding upon which scale the answer is to be found. In this connection it is helpful to bear in mind the relationship between the upper and lower log-log-scales, since, if the index of the root be less than 10, it will be clear that the answer must be larger than the tenth root, as read on the upper log-log scale; with a larger index than 10, the value of the root will obviously be less than that of the tenth.

EXAMPLE 12. $\sqrt[7]{950} = 2{\cdot}66.$

Bring 7 on C in line with 950 on the log-log scale, and read 2·66 on the lower log-log scale in line with the left hand index of C. In this particular example the root may also be read on the upper log-log scale above the right hand index of C.

EXAMPLES 13. (a) $\sqrt[1{\cdot}6]{62} = 13{\cdot}2.$

(b) $\sqrt[0{\cdot}8]{252} = 1004.$

(c) $\sqrt[12]{38} = \sqrt[1{\cdot}2]{\sqrt[10]{38}} = 1{\cdot}354.$

$\sqrt[10]{38}$ is found on the upper log-log scale over 38 on the lower; without troubling to read this, bring 1·2 in line with it and read 1·354 on the upper log-log scale, above the left hand index of C. In example 13 ⟨b⟩, 8 on C is brought against 252 on the lower log-log scale, and the answer, 1004, is read under the right hand index of C.

Exponential equations of the form $a^x = b$, where a and b are known, may be solved by reversing the process of involution; set the index of scale C to a on the log-log scale, and, opposite b on the log-log scale, read x on C. When b exceeds a, the exponent x is larger than 1, and a and b may be together on the upper scale, or lower scale, or a may be on the upper and b on the lower scale. In the latter case, the position of b is known if $x = 10$, and from this we may see whether x is greater or less than 10 — if b, on the lower scale, lies to the right or left of a on the upper.

Examples 14. ⟨a⟩ $1{\cdot}25^x = 2{\cdot}3$, $x = 3{\cdot}74$.
⟨b⟩ $2{\cdot}1^x = 9{\cdot}3$, $x = 3$.
⟨c⟩ $1{\cdot}185^x = 2920$, $x = 47$.
⟨d⟩ $5{\cdot}5^x = 1000$, $x = 4{\cdot}05$.
⟨e⟩ $15{\cdot}5^x = 30000$, $x = 3{\cdot}76$.

When a exceeds b, the exponent x is less than 1. Both a and b may be on the upper scale, or lower scale, b being to the left of a. Also b may be on the upper scale and a on the lower, in which case, as b lies to the right or left of $\sqrt[10]{a}$, so x is above or below 0·1.

Example 15. $1{\cdot}585^x = 1{\cdot}1525$, $x = 0{\cdot}308$.

Set the right hand index of C to 1·585 on the upper log-log scale and read 0·308 on C, under 1·1525 on the upper scale.

Examples 16. ⟨a⟩ $20000^x = 2{\cdot}69$, $x = 0{\cdot}1$.
⟨b⟩ $12^x = 1{\cdot}95$, $x = 0{\cdot}269$.

Set the left hand index of C to 12 on the lower log-log scale, and under 1·95 on the upper scale read 0.269 on C.

⟨c⟩ $20000^x = 1{\cdot}104$, $x = 0{\cdot}01$.

Set the right hand index of C to 20000, and under 1·104 read 0·01 on C.

The efficiency and Voltage Drop Scales.

On the bottom of the slot where the slide moves, two scales will be found, the one graduated in black giving the efficiencies of dynamos and motors, and the red one giving the voltage drop for electrical conductors. Each of these scales is read against the metal pointer fixed to the left hand end of the slide.

The efficiency scale is used with scales A and B, and for this purpose A represents kilowatts and is marked KW at the right hand end, while horsepower is taken on B, which is marked H. P.

By setting the metal index pointer to 100 per cent efficiency, we find that 10 horsepower — the left hand index of B — is against 7·46 kilowatts on A, and with this setting the kilowatts on A corresponding to any horsepower on B, or vice versa, may be read off. To the left and right of this point — marked 100 — will be found the efficiencies of dynamos and motors down to 20 per cent.

When the metal pointer is against the efficiency on the dynamo scale, the input in horsepower on scale B is against the output in kilowatts on A for all inputs and outputs at this efficiency. In the same manner, when the pointer is set to an efficiency on the motor scale any value of input in kilowatts on scale A will be found against the output in horsepower on scale B.

Example 17. Determine the kilowatts taken by a motor which develops 620 horsepower, the efficiency being 95 per cent.

Set the metal pointer to 95 per cent — to the left of 90 — on the motor scale and against 620 HP on B read 487 KW on A.

Example 18. Determine the horsepower required to drive a dynamo which is 82 per cent efficient, and supplies 75 kilowatts.

Set the pointer to 82 per cent on the dynamo scale and against 75 kilowatts on A read 123 horse power.

Example 19. A three phase induction motor develops 50 horsepower, and is supplied with a line current of 125 amperes at 250 volts, the power-factor being 0·8. Determine the efficiency.

The input for a three phase motor is $\sqrt{3}$ E.I.Cos $\theta = \sqrt{3} \times 250 \times 125 \times 0{\cdot}8$. Set the cursor to 3 on A and read 1·732 on D. Set the left hand index of B to 1·732 on A, move the cursor over 250 on B, bring the left hand index of B under the cursor line, move the cursor over 125 amperes on B, bring the left hand index of B to the cursor line and move the cursor over 0·8. Now bring 50 horsepower on B under the cursor line and read 86 per cent on the motor efficiency scale.

The scale, giving the loss of potential in copper conductors on direct current circuits, or alternating current circuits of unity power factor, lies parallel to the efficiency scale, and is marked in red, thus conforming to the colour of the index marks relating to this scale on A and B. These marks, "10 Amp.", "10 yd.", and "10000 circ. mil.", on the face of the rule and slide mean that 1 on A must be read as 10 amperes, and 1 on B as 10 yards and 10,000 circular mils when using the voltage drop scale. From this it will be seen that current is taken on A, and length and cross-section are taken on B.

The voltage drop scale is based on the formula: $e = \frac{I \times L}{c \times a}$, where I = current in amperes,

L = total length of conductor in yards,

c = 0·0327 mho, volume conductivity of copper (mil, yard) at 60° F.

a = area of cross-section of conductor in circular mils.

In using the scale, the current and the length of the conductor are multiplied together on the A and B scales, the area of cross-section of the conductor on B is brought to the product of these two on scale A, and the voltage drop is read against the metal pointer on the red scale. The scale is so graduated that division by c is not necessary.

To find the area of cross-section in circular mils, it is only necessary to square the diameter in mils — i. e., in thousandths of an inch. For instance, a wire of diameter 0·128 inch, or 128 mils, is $128 \times 128 = 16{,}400$ circular mils in area.

Example 20. Determine the voltage drop across the ends of a copper conductor 500 yards long and 41,000 circular mils in area when the current is 12·9 amperes.

Set the left hand index of B to 12·9 amps. on A (taking the left hand index of A as 10 amperes), move the cursor over 500 yds. on B (the left hand index of B being 10 yds., 500 yds. will be the graduation marked 50 on the right hand B scale), bring 41,000 circ. mils on the left hand B scale under the cursor, and read 4·8 volts against the pointer on the voltage drop scale.

Example 21. A copper conductor 200 yards long and 0·160 inch in diameter carries 40 amperes, find the loss of potential.

The diameter being 160 mils, the area will be 160^2 or 25,600 circ. mils.

Set 1 on B to 40 amps. ⟨4⟩ on A, move the cursor over 200 yds. ⟨20⟩ on B, bring 25,600 circ. mils ⟨2·56⟩ on B under the cursor, and read 9·5 volts, the required answer, against the metal pointer.

The voltage drop scale gives the result with the decimal point in the right position so long as the values set on the A and B scales have their decimal points correctly placed in relation to the left hand indices of these scales. For instance, the graduation marked 4 on the left hand B scale must be taken as 40 amperes, but it could not be used for 4 or 400 amperes without making an adjustment in the answer. Similarly, the graduation 20 on the right hand B scale is 200 yards or 200,000 circular mils, and would not produce an answer with the decimal point correctly placed if used for 2,000 yards or 20,000 circular mils. It is, however, always possible to bring the current, length and sectional area within these limits by multiplying or dividing by a multiple of 10 and adjusting the voltage that is read from the voltage drop scale accordingly.

Example 22. What voltage is required to send 100 amperes through 1,500 yards of wire 0·432 inch in diameter?

The area in circular mils ⟨432^2⟩ need only be found to two or three figures; it is 187,000.

$$\text{The required voltage } e = \frac{100 \times 1{,}500}{c \times 187{,}000} = \frac{100 \times 15}{c \times 18{,}700} \times 10.$$

Set 1 on B to 100 amps. ⟨10⟩ on A, move the cursor over 15 yds. ⟨1·5⟩ on B, bring 18,700 circ. mils ⟨1·87⟩ on B under the cursor, and read 2·45 on the voltage drop scale. Since this must be multiplied by 10, the answer is 24·5 volts.

Example 23. Calculate the voltage drop across 2,500 yards of wire which is 0·080 inch in diameter and carries 6 amperes.

$$e = \frac{6 \times 2{,}500}{c \times 80^2} = \frac{6 \times 2{,}500}{c \times 6{,}400} = \frac{60 \times 25}{c \times 64{,}000} \times 100.$$

Set 1 on B to 60 amps. ⟨6⟩ on A, move the cursor over 25 yds ⟨2·5⟩ on B, bring 64,000 circ. mils ⟨6·4⟩ on B under the cursor, and read 0·715 volts on the voltage scale. As this must be multiplied by 100, the answer is 71·5 volts.

Should the voltage obtained be too large, it is only necessary to move the slide until the pointer is on the correct value, when

under the cursor line on scale B will be found the corresponding cross-section of conductor.

Example 24. Determine the potential across the ends of 2,000 yards of a wire of 25,600 circular mils cross-section, the current being 40 amperes.

Set 1 on B to 40 amps. on A, move the cursor over 200 yards on B, bring 25,600 circ. mils on B under the cursor, and read 9·5 volts against the pointer. The length was taken as 200 yards instead of 2,000, and so the above reading must be multiplied by 10. The answer is, therefore, 95 volts.

Had the maximum voltage drop been limited to 35 in the last example, the required cross-section of conductor may be found by leaving the cursor over 25,600 circ. mils on B, with the pointer against 9·5 volts, and moving the slide until the pointer indicates 35 volts ⟨3·5, owing to 200 yds. being taken⟩. Under the cursor on B will now be found 70,000 circ. mils nearly. The square root of 70,000 will give the diameter in mils of the wire required. This diameter is 264, or 0·264 inch, which is not a stock size, the nearest being 0·276 inch. A wire 276 mils in diameter has a cross-sectional area of 76,200 circ. mils. Leave the cursor over 70,000 on B, bring 76,200 on B under it, and read 32 volts, the loss of potential over 2,000 yds. of 0·276 inch wire when 40 amps. are flowing. To find the total length of such a conductor for a loss of 35 volts, set the pointer to 35 volts, put the cursor over 76,200 circ. mils on B, move the left hand index of B to the current ⟨40 amps.⟩ on A and read 2,200 yards, the required length, on scale B under the cursor line. In this way, the length corresponding to any current may be found, or the current flowing through any length of conductor for a given loss of potential is found by setting the length on B under the cursor and reading the current on A over the left hand index of B.

The currents and lengths of wire may be conveniently compared by setting the pointer to the given potential difference across the ends of the wire, moving the cursor over the cross-section of wire on B, inverting the slide and bringing 1 on BR under the cursor line. Amperes will now be on A over the corresponding lengths on BR. With a pressure drop of 35 volts and a conductor of 76,200 circular mils area, the following readings were obtained:

Under	80	amps.	on	A	read	1,100	yds.	on	BR,
"	50	"	"	A	"	1,760	"	"	BR,
"	35	"	"	A	"	2,500	"	"	BR,
"	25	"	"	A	"	3,500	"	"	BR,
"	10	"	"	A	"	8,800	"	"	BR,
"	5	"	"	A	"	17,600	"	"	BR.

It will be understood that 5 amps. and 50 amps. are both read on the same graduation.

Example 25. An electro-magnet is wound with 820 yards of copper wire 0·0124 inch in diameter; determine the E. M. F. when 0·12 amp. flows through the coil.

$$e = \frac{0{\cdot}12 \times 820}{c \times 12{\cdot}4^2} = \frac{12 \times 10^{-2} \times 82 \times 10}{c \times 15{,}400 \times 10^{-2}} = \frac{12 \times 82}{c \times 15400} \times 10.$$

Set 1 on B to 12 amps. on A, move the cursor over 82 yds. on B, bring 15,400 circ. mils under the cursor, and read 1·95 volts on the volt scale. As shown above, this has to be increased ten-fold; the answer is, therefore, 19·5 volts.

Example 26. Determine the current in the last example when the pressure is 24 volts.

Set the pointer to 2·4 on the volt scale, move the cursor over 15,400 circ. mils on B, bring 82 yds. on B under the cursor, and read 0·148 amps. on A over the left hand index of B.

The size of the answer in Example 26 should be obvious, but this may be always checked by setting the problem out as follows:

$$I = \frac{24 \times 154}{820 \times c} = \frac{2{\cdot}4 \times 10 \times 15{,}400 \times 10^{-2}}{82 \times 10 \times c} = \frac{2{\cdot}4 \times 15{,}400}{82 \times c} \times 10^{-2}.$$

The slide rule reading was 14·8 amps. which with the decimal point moved back two places gives the answer 0·148 amps.

Actually, this can all be done mentally: the voltage was taken one-tenth of that given, whence the current is to be increased ten-fold; the area was 100 times too large, whence the current is to be decreased a hundred-fold; the length was taken as one-tenth of that given, therefore, the current is to be again divided by 10. The reading, 14·8, must, consequently, be $\frac{14{\cdot}8 \times 10}{100 \times 10} = 0{\cdot}148$ amps.

Gauge Points W and R.

These gauge points are used for calculating the weights and resistances of copper conductors. W relates to weight and R to resistance. The positions of these points are so chosen that, by taking the length in yards and the diameter of the wire in mils., the weight will be given in pounds and the resistance will be that of an annealed copper conductor in ohms at 60° F.

The gauge point W is taken at 332 on scale D and its square on scale A is nearly 110000. In using this gauge point, we get $\frac{d^2}{w} = \frac{W^2}{L}$ where $d =$ diameter of wire in mils,

$L =$ length in yards,
$W = 332$,
$w =$ weight of wire in pounds.

This becomes $\frac{d^2}{w} = \frac{110000}{L}$.

As 1 mil. = 0·001 inch, mils. are found directly from the diameter of the wire in decimals of an inch. For instance, 0·16″ = 160 mils.

To find the weight of a copper wire, put the length on scale B in line with the gauge point *W*, move the cursor over the diameter on scale D and read the weight on scale B under the cursor.

Example 27. Determine the weight of 640 yards of copper wire 0·048 inch in diameter.

Set 640 ⟨6·4⟩ yards on B to *W* and over 48 ⟨4·8⟩ mils. on D, read 13·4 pounds on B.

Gauge point *W* applies only to copper wires, but no difficulty will be experienced in finding the weights of wires of other metals by using the weights of metals table on the back of the slide rule. The weight of the same diameter and length of copper is determined as above explained and multiplied by

$$\frac{\textit{weight of 1 cu. ft. of required metal}}{\text{weight of 1 cu. ft. of copper.}}$$

These values are obtained from the table on the back of the rule.

Example 28. Find the weight of 970 yards of an aluminium wire which is 0·128 inch in diameter.

Set 970 on B to *W* and over 128 on D read 144 pounds on B. This is the weight of a copper wire of the above dimensions. From

the above mentioned table, we find that a cubic foot of copper weighs 552 pounds and a cubic foot of aluminium weighs 166 pounds. Set 552 on B to 144 on A and over 166 on B read 44 pounds, the required weight, on A.

Gauge point R is taken at 5·527 on scale D and when the slide is set, we get $\frac{d^2}{L} = \frac{R^2}{r}$ where $d =$ diameter of wire in mils.
$L =$ length in yards.
$R = 5{\cdot}527.$
$r =$ resistance in ohms.

In using this gauge point, we set the length of conductor on scale B to the diameter on scale D and above gauge point R read the resistance on B.

Example 29. Determine the resistance of 250 yards of copper conductor 0·128 inch in diameter.

Set 250 ⟨2·5⟩ yards on B to 128 ⟨1·28⟩ mils. on D, move the cursor over R and read the resistance, 0·466 ohms, on B under the cursor line.

The slide occasionally has to be set so far to the left that scale B is no longer over gauge point R. In this case, the length on B is set to the diameter on D, the cursor is put over the right hand index and the left hand index is brought under the cursor. The resistance can then be read over R as before.

Example 30. A copper conductor is 580 yards long and its diameter is 0·104 inch. What is its resistance?

Set 580 yards on B to 104 mils on D, move the cursor over the right hand index of B, bring the left hand index under the cursor and read 1·64 ohms on B above gauge point R.

This gauge point is only suitable for copper, but by using the table of comparative resistance on the back of the rule, the resistances of other metals may be obtained. The resistance of copper is taken as 1 in the table, and alongside the other materials will be found factors by which the resistance of copper must be multiplied to give the resistance of the required metal. In calculating, the gauge point is used as for a copper wire of the necessary dimensions, and the answer is multiplied by the factor given.

Example 31. Find the resistance of the wire in Example 30 if composed of ⟨a⟩ iron, and ⟨b⟩ aluminium.

The resistance of the copper wire was 1·64 ohms, and the table gives 7·5 and 1·6 for iron and aluminium respectively. The left hand index of B is set to 1·64 on scale A and, against 7·5 and 1·6 on B, the answers, 12·3 and 2·62 ohms respectively, are found on scale A.

Should there be any uncertainty as to the position of the decimal point in answers obtained with these gauge points, the size of the result may be simply estimated from the equations given. For instance, in Example 27, we have

$$\frac{48^2}{w}=\frac{110000}{640},$$

roughly this may be taken as $\frac{50^2}{w}=\frac{110000}{600}$.

This cancels to $\frac{25}{w}=\frac{11}{6}$,

and $11\,w = 150$,

from which $w = 13{\cdot}6$.

Temperature-Resistance Scale for Copper Wire.

On the slide, just to the right of the centre, will be found a short scale marked in degrees Fahrenheit. This scale which is used in combination with scale A, permits the variation of the electrical resistance of copper wires to be compared.

When the resistance of a wire at any of the marked temperatures is known, the resistance at another temperature may be found by placing the known temperature on the temperature scale in line with the known resistance on A and reading the required resistance on A over the corresponding temperature graduation.

Example 32. The resistance of a copper wire is 20 ohms at 59° F., find its resistance at 77° and 32°.

Set 59° on the temperature scale in line with 20 ohms on A and, with the aid of the cursor, read 18·8 ohms and 20·8 ohms respectively on A over 32° and 77°.

Example 33. A coil has a resistance of 30 ohms at 60° F., find its resistance at 100°.

Place the cursor line over 30 ohms on A, set the slide so that 59° on the temperature scale is just to the left of the cursor line, move the cursor over 100° (almost exactly midway between 77° and 122°) and read 32·7 ohms.

The graduations on this scale are the Fahrenheit equivalents of 0°, 15°, 20°, 25°, 50° and 75° Centigrade.

The A.W. FABER "CASTELL" Slide Rule No. 398

(10 in. scale length).

This also applies to Slide Rule No. 319

(5 in. scale length).

For Electrical and Mechanical Engineers.

This slide rule is equipped with the same scales as the one which was described in the foregoing pages; it also carries a reciprocal scale and a scale of cubes, the latter being placed on the lower edge of the rule body. Owing to the similarity between these two rules, the instructions for the No. 378 Slide Rule apply to all scales in this case, except the reciprocal and cube scales, which will now be explained. It will, therefore, be understood that the instructions for No. 398 Slide Rule commence on Page 55 and are continued on this page.

The reciprocal scale, which is on the face of the slide, commences at the right hand end with the graduation 0·89 and finishes at 11·2 at the left hand end of the slide, the indices at both ends coinciding with the indices of B and C. Since this scale is identical with the C scale, only running in the reverse direction, it will be called CR.

The reciprocals of numbers on C will be found on CR above the numbers, and may be read off with the aid of the cursor. For instance, above 3 on C read 0·333 on CR, above 2 read 0·5, and above 7 read 0·143.

The location of the decimal point in the reciprocal is found by the rule already given for division: the number of figures will be 1 — N, where N is the number of figures in the number.

EXAMPLES 34. $\frac{1}{26} = 0{\cdot}0385, \frac{1}{73} = 0{\cdot}0137, \frac{1}{12} = 0{\cdot}0833.$

Example 35. Determine the joint resistance of three parallel electrical circuits 7·5 ohms, 16 ohms and 22 ohms resistance.

$$R = \frac{1}{\frac{1}{7{\cdot}5} + \frac{1}{16} + \frac{1}{22}} = \frac{1}{0{\cdot}133 + 0{\cdot}0625 + 0{\cdot}0455} = \frac{1}{0{\cdot}241}.$$

The above values are read on CR above 7·5, 16, and 22 on D. The reciprocal of 0·241 is read on CR above the number on D; it is 4·15 ohms, the required resistance.

When the reciprocal of the square root of a number is required, the cursor is set over the number on B and the reciprocal is read on CR under the cursor.

Examples 36. $\frac{1}{\sqrt{3}} = 0{\cdot}577$, $\frac{1}{\sqrt{22}} = 0{\cdot}213$, $\frac{1}{\sqrt{236}} = 0{\cdot}065$.

The reciprocal of the square of a number is found by putting the cursor over the number on CR and reading the answer above on B.

Examples 37. $\frac{1}{2{\cdot}32^2} = 0{\cdot}186$, $\frac{1}{6{\cdot}4^2} = 0{\cdot}0244$, $\frac{1}{7{\cdot}2^2} = 0{\cdot}0193$.

The cube scale CU is used in combination with scale D. Scale CU gives the cubes of the numbers on scale D in three separate groups, thus allowing the number of figures in the cubes to be found without the use of rules. Numbers between 1 and 10 have cubes between 1 and 1,000, and as the first group contains the cubes of numbers between 1 and $2{\cdot}1544 = \sqrt[3]{10}$, the second between 2·1544 and $4{\cdot}6416 = \sqrt[3]{100}$, and the third between 4·6416 and $10 = \sqrt[3]{1000}$, we may read the cube and fix the number of figures at once, the first group containing units, the second tens, and the third hundreds. By moving the decimal point in the number to be cubed if it exceeds 10, it may be brought on to the D scale and the cube found, the decimal point being then moved back three times the number of places it was previously moved in the opposite direction.

Example 38. $1{\cdot}8^3 = 5{\cdot}83$.

Place the cursor over 1·8 on D and read 5·83 on CU.

Example 39. $3{\cdot}6^3 = 46{\cdot}7$.

Move the cursor over 3·6 on D and read 46·7 on CU.

That this answer is 46·7, and not 4·67. is evident from the fact that it is read on the intermediate portion of the cube scale.

EXAMPLE 40. $6{\cdot}8^3 = 314$.

In line with 6·8 on D read 314 on the cube scale, this reading being in the third group.

EXAMPLE 41. $18^3 = 5{,}830$.

This may be changed to $1{\cdot}8^3 \times 10^3$, when the cube of the first factor, 5·83, will be found in the first group. This must be multiplied by 1,000, as shown by the second factor, and the answer is 5,830.

EXAMPLE 42. $0{\cdot}014^3 = 0{\cdot}00000274$.

This should be taken as $1{\cdot}4^3 \times 10^{-2\times3}$. The cube of 1·4 is found in line with 1·4 on D and is 2·74. The decimal point must be moved back six places to the left, which makes the answer 0·00000274.

In cube root, the number is pointed off in periods of three figures from the decimal point, and as the left hand period contains one, two, or three figures, so the number is taken on the left hand, intermediate, or right hand portion of the cube scale. One figure is required in the answer for each period thus pointed off.

EXAMPLE 43. $\sqrt[3]{9{\cdot}6} = 2{\cdot}125$.

Since the number contains one figure, the left hand portion of CU must be used. Set the cursor extension line to 9·6 on the cube scale and read 2·125 on D.

EXAMPLE 44. $\sqrt[3]{35} = 3{\cdot}27$.

The number is taken on the middle portion of the cube scale.

EXAMPLE 45. $\sqrt[3]{176} = 5{\cdot}6$.

This must be taken on the right hand portion of CU, since it contains three figures.

EXAMPLE 46. $\sqrt[3]{0{\cdot}0042} = 0{\cdot}1613$.

Calculate as $4{\cdot}2^{\frac{1}{3}} \times 10^{-\frac{3}{3}}$. The number is taken on the left hand portion of CU, and the answer is $1{\cdot}613 = 10^{-1} = 0{\cdot}1613$.

When reciprocals of cube roots are required, the line on the cursor is set against the number on CU and, with the indices of CR and D in line, the answer is read on CR under the cursor line.

EXAMPLE 47. $\frac{1}{\sqrt[3]{24}} = 0{\cdot}347.$

Set the cursor line against 24 on the middle group of CU and read 0·347 on CR.

The reciprocal of a cube is found by setting the cursor over the number on CR and reading the answer against the cursor line on CU, the indices of CR and D being in line.

EXAMPLE 48. $\frac{1}{2{\cdot}5^3} = 0{\cdot}064.$

Two-third powers, such as $354^{\frac{2}{3}}$, are simple solved with scale CU, since they can be put into the form $\sqrt[3]{354^2}$. To solve this, set the cursor line over 354 on the hundreds portion of the cube scale and read the answer, 50, on A under the cursor.

Three-half powers are obtained by setting the cursor over the number on scale A and reading the answer on the cube scale. It is advisable to point the number off for square root before setting on scale A. To find the value of $730^{\frac{3}{2}}$, point the number into 7 30, place the cursor over 7·3 on scale A and read 19·7 on the CU scale, tens portion. As there were two groups in the number, the decimal point must be shifted three places to the right, since two "cube" groups are required. The answer is 19,700.

Multiplication may be done with scales CR and D by bringing one factor on CR to the other factor on D and reading the product on D against the index of CR. Since the slide is now in position for multiplying the product of the two factors by a third factor taken on C, it follows that by using scales CR, C, and D we may find the product of three factors with one slide setting, provided the third factor on C does not happen to be beyond the end of the rule body.

EXAMPLE 49. $16 \times 44 \times 14{\cdot}8 = 10{,}400.$

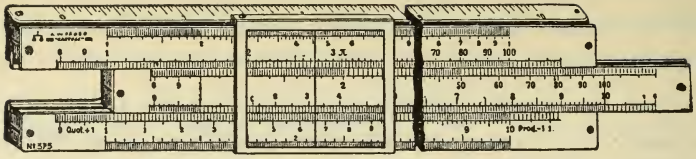
Set 44 on CR to 16 on D and against 14·8 on C, read 10,400 on D.

Square roots may be found by means of scales CR and D, and since these scales are more openly spaced than A, the roots are more easily read.

To find the square root of a number, set the index of CR to the number on D, using the left hand index for even numbers of digits and the right index when the number of digits is odd, and move the cursor along until the same number is found on CR and D, this number is the square root.

EXAMPLE 50. $\sqrt{5270} = 72{\cdot}6.$

Set the left hand index of CR to 5270 on D and move the cursor along until the same reading, 72·6, is found simultaneously under the cursor line on CR and on D.



The A.W. FABER "CASTELL" Slide Rule No. 375. With Scale for Cubes and Cube Roots.

This slide rule, in addition to scales A, B, C and D, carries a scale of cubes above the A scale, and the L scale for logarithms has been placed on the front of the rule below scale D, leaving a space on the back of the slide upon which an extra scale has been engraved, this scale being used in finding sines and tangents of small angles.

The method of using the cube scale will be understood from the explanation commencing on the middle of Page 70 and continuing on Page 71.

The scale of logarithms on the lower edge of the rule requires no explanation as it is read exactly as described in the general instructions for the L scale with the slide reversed.

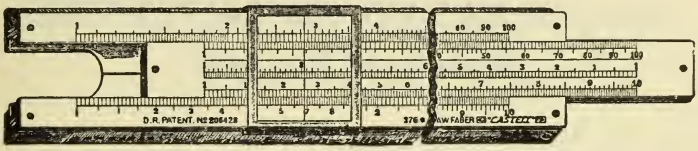
The trigonometrical scales S and T on the back of the slide on this rule are graduated so as to give both sines and tangents on scale D.

If the slide be reversed in the rule body so that scale T is against D and the indices of A, S, T and D are in line at both ends, then opposite any angle on S will be found its sine on D; the significant figures following immediately after the decimal point. When the angle is below 5^0 $40'$ the scale marked S & T is used, the sine and tangent being found on D in line with the angle on S and T, one cypher following the decimal point. For angles less than 6^0 the sines and tangents are interchangeable, the first three figures at least being the same, and for smaller angles the difference is extremely small.

With the slide in its normal position tangents are found as described in the general instructions, and sines are obtained by setting

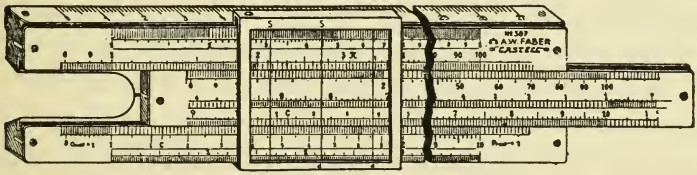
the angle on S to the S index line at the back of the rule and reading the sine on scale C against the D index.

The angle on the S & T scale is brought against the lower index line on the back of the rule at the right hand end, and the sine or tangent is read on C over the right hand index of D. The sines and tangents found with this scale are between 0·01 and 0·1.



The A.W. FABER Polyphase Slide Rule No. 376.

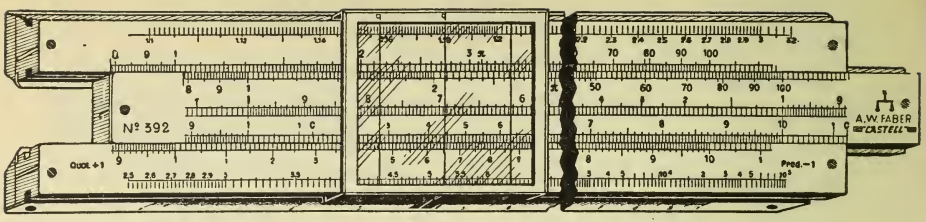
This slide rule is fitted with the usual scales on the face, as well as a reversed C (Reciprocal) scale along the centre of the slide and a scale of cubes on the lower edge of the rule body. The instructions for using these two scales will be found on Pages 69, 70, 71, and 72.



The A.W. FABER "CASTELL" Slide Rule No. 387.

This slide rule is fitted with a scale for cubes and cube roots above the A scale, and the scale of logarithms is engraved below scale D on the face of the rule. The inverted C scale, CR, is arranged along the centre of the slide face, while a scale, S & T, for reading the sines and tangents of small angles, is placed upon the back of the slide.

The manner in which these scales are employed will be understood from the instructions given on Pages 69 to 73 inclusive.



The A.W. FABER "CASTELL"
Slide Rule No. 392.

This rule has, in addition to the usual Square, Square Root, Sine, Tangent and Logarithm scales of an ordinary rule, a **Log-Log scale** and a **Reciprocal Scale** on the face of the rule with a **Cube Scale** along the lower edge.

Apart from the usual slide rule calculations, the additional scales render it possible to find the following —

$$a^3,\ 1 \div a^3,\ \sqrt[3]{a},\ 1 \div \sqrt[3]{a},\ a^{\frac{2}{3}},\ a^{\frac{3}{2}}$$

without moving the slide. These powers can also be calculated in conjunction with the other terms.

RECIPROCAL values can be read without movement of the slide, whilst products of three factors and divisions by two divisors require only one movement of the slide. — Quadratic and cubic equations can be easily solved.

Invaluable for Mechanical Engineers. — Brief instructions are given with each rule.

THE LIDDERDALE SLIDE RULE

1. Hints for usage

If one is not used to a slide-rule, a little practice is necessary to acquire dexterity and to familiarise oneself with the method and with the divisions on the scales, but calculation soon becomes automatic. Make use of the cursor as much as possible, as this saves eye-strain and increases accuracy.

Always keep one jump ahead of the slide-rule calculation by making a very rough estimate in your head. The easiest mistake is to read a result ten times too much or too little, or to set say 8½″ instead of 8′ 6″.

Speed with accuracy very largely depends on the exact way in which one controls the slide and cursor. The following hints are offered. Pull out the slide to about ½″ short of the final position, then change your grip so that the thumb is pressed against the stock, underneath the slide and the fingers have an overhand grip. Then squeeze the slide into its exact position. Next shift the cursor to approximately its final position, then place the tip of each thumb against the bottom corners of the cursor whilst the fingers grip the rule. It is then much easier to squeeze the cursor into its exact position, as compared with holding the rule with one hand and moving the cursor with the other.

The examples given below refer mostly to problems encountered by builders and flooring contractors, but the same basic methods of calculation apply to very many uses.

2. Areas

For calculating areas, use scales A and B together. Only use scale L when calculating volumes. It does not matter whether Width is set on A and Length on B or vice versa.

To find the area of a room 20′ 6″x9′ 4″, set the cursor line over 20′ 6″ on A and then move the slide from left to right until 9′ 4″ is under the line. (Remember that on scale B the inches are to the *left* of the feet marks, so be careful not to set at 9′ 8″ by mistake.) Then move the cursor until the line is over 1 on C and read 21.3 (say 21¼) sq. yds. on D or 192 sq. ft. on F.

If the area is large and one dimension is greater than 30′, the easiest way is to halve this and then double the answer. For 48′ 6″x28′ 3″, for instance, set 24′ 3″ over 28′ 3″, read 76.2 sq. yds. and double to find the answer is 152½ sq. yds.

If the area is less than 1 sq. yd., 1 on C will run off the D scale to the left, but one can continue to read under 100 on C bearing in mind that the D scale now represents 0.01 to 1.00 instead of 1 to 100 sq. yds. Thus 21 lineal feet of 4″ Strip is 0.78 sq. yds. or 7.0 sq. ft.

Many rooms have projections and alcoves. These small areas can be quickly calculated and then added or deducted from the main areas.

The rule can, of course, be used to find the area of a roll of sheet goods. If the length is more than 30′, set width at 12′ for 6′ material and halve the length. For two-metre wide material, set at 6′ 6¾″ instead of 6′.

To calculate the area of a circle, set half the diameter (i.e. the radius) on A and half the diameter on B. Then read the area under π (at 3.14 on C) in sq. yds. on scale D or sq. ft. on scale F. Thus the area of a 15′ 6″ circle is 7′ 9″x7′ 9″x3.14 sq. ft., that is, 189 sq. ft. or 21 sq. yds.

It will be noted that by using scales D and F together, sq. ft. can be instantly converted to sq. yds. and vice versa.

3. Waste

Having found the area of a room, it may be necessary to allow for waste. This is extremely simple to calculate on the slide-rule. Suppose the area is 58.2 sq. yds. Set 1 on C over 58.2 on D. If you move the cursor from 1.0 on C to 1.1, then the reading on D below the line will be the square yards at C 1 multiplied by 1.1 or 110/100: that is, increased by 10%. Supposing you envisage 5% waste. Move the cursor to 1.05 and read 61.1 sq. yds. If you have to buy tile in 5 yard cartons, you must order either 60 or 65 sq. yds. Move the cursor to 60 on D and read 1.03 on C and to 65 and read 1.12. Twelve cartons will therefore allow only 3% for waste whereas thirteen will give 12%.

4. Volumes

To find the capacity of a room in cu. ft., set Height on B under Width on A, then move cursor to Length on L and read the answer under the cursor line on F.

For example, a room is 20′ 9″x13′ 5″x10′ 10″... Set 13′ 5″ on A over 10′ 10″ on B. The Length at 20′ 9″ on L is now off the right-hand end of the scales. Therefore set the cursor over 1 on L and push back the slide so that 100 on L comes under the line. (All that part of scale L which was beyond the others will now be available, but after resetting the slide in this way one must remember to multiply the answer by 100.) Now move the cursor to 20′ 9″ and read the answer as 30.3x100, or 3030 cu. ft. on F, or 1.12x100, or 112 cu. yds. on V.

Similarly, to calculate the quantity of concrete required for a slab or wall 66′ x18′ 6″x5″, set 18′ 6″ on A over 5″ on B, move cursor to 66′ on L and read 18.9 cu. yds. on V.

To convert cu. ft. to gallons, multiply by 6.2 for Imperial and 7.5 for U.S. gallons (see §10 below).

5. Mixing Proportions

Proportions for mixing say concrete can be quickly calculated after finding the total volume required.

For a 1:2:4 mix, for instance, it is assumed that the proportions will be 520 lbs. of cement, 0.45 cu. yd. of sand and 0.86 cu. yd. of coarse aggregate. It will probably be more convenient to call the cement 5.20 bags of 100 lbs. or 4.64 bags of 112 lbs. (Standard factors such as these can be noted on the back of the rule by roughening the surface with a typewriter eraser and writing with waterproof ink.)

With the cursor over 18.9 cu. yds. on V, move the slide back so that 10 on C is under the line, then move the cursor to 4.64 on C and read on V 8.8x10, that is, 88 bags of 112 lbs. of cement. (Alternatively, over 5.20 on C read 9.8x10 100-lb. bags.) Then move the cursor to 4.5 (i.e. 0.45x10) on C and read 8.5 cu. yds. of sand on V, and to 8.6 on C and read 16.3 cu. yds. of coarse aggregate on V.

6. Tiles per Dimension

Simply to guess a waste figure on a percentage basis is wasteful. The slide-rule can be used to determine exactly how many tiles are required to fit into a given length or breadth.

(a) Square Designs

No problem arises with 12″x12″ tile. If a room is 23′ 3″ long, one knows that 24 tiles will be required, giving a 7½″ border tile at each end.

It is not so easy to calculate 9″x9″ tile in one's head. Therefore set 75 on C over 100 on D. Now by setting the cursor over any dimension on L, the number of 9″x9″ tiles can be read on D. Thus 24′ 8″ is equivalent to 32.9 tiles. This is too near 33 for safety, in case the walls run off-square. Therefore 34 tiles are required, giving approximately 5″ border tiles.

Supposing that one had calculated 34x28 tiles, to find the yardage required, set the cursor over 3.4 on D and set 1 on C above it. Then move the cursor to 2.8 on C. Then, since there are 16 9″x9″ tiles per square yard, draw the slide until 1.6 comes under the cursor. Read 5.9 (5.925) on D under 1 on C. Obviously the answer is neither 5.9 nor 590 sq. yds., so it must be 59 (59¼).

(b) Diagonal Designs

For exact size tiles, diagonal dimensions are:

$$9'' = 12\tfrac{22}{32}'' \text{ or } 1.06'$$
$$12'' = 16\tfrac{31}{32}'' \text{ or } 1.42'$$
$$18'' = 25\tfrac{15}{32}'' \text{ or } 2.12'$$

Therefore, to find the number of 12″ tiles to be laid diagonally within a length of 26′ 5″, set 1.42 on C over 1 on D and read 18.6 tiles on D under 26′ 5″ on L.

To find the number of tiles required to cover the area, multiply the number in Length by the number in Width and then *double* the answer.

7. Repeats

It is easy to calculate how many carpet or wallpaper widths, or pattern repeats, go into a given dimension. Simply set the size of repeat on L over 1 on D, move the cursor to the room dimension on L and read the number of repeats on D.

For instance, for a carpet width or repeat of 27″, set cursor over 1 on D and move slide until 2′ 3″ is under line. for a dimension of 18′ 3″, move cursor to 18′ 3″ on L and read 8.1 on D, thus showing that 9 widths or repeats are required.

8. Strip

To find the lineal footage of strip to cover a floor of given area, simply set the cursor over 108 on F, move the slide so that the strip width in inches on C is under the cursor line; then move the cursor so that it is over the square yardage on C and read the lineal footage on F.

For example, to find the number of lineal feet of 1¾″ strip to cover a floor of 12¼ sq. yds., set 1.75 on C over gauge-mark LF at 108 on F, then move cursor to 12.25 on C and read 756 lineal feet on F. (For 36 yards, move to 3.6 on C and read 222x10 lin. ft.)

Conversely, to find the area of 1,650 lin. ft. of 2⅝″ strip, set 2.625 on C over 108 on F, move cursor to 165 on F and read 4.01x10, or say 40 sq. yds., on C.

9. Units per Square Yard

To calculate the number of units of a given size which go to a square yard, simply set Width under Length in the normal way and read the number of tiles on C above 1 on D.

For example, for 10½″x10½″ tiles, set 10½″ on B under 10½″ on A and read 11.75 tiles per square yard on C. To find the number of tiles for 18½ sq. yds., move the cursor to 1.85 on D and read 218 (21.75x10) tiles on C.

For wood blocks say 3⅛″x9$\frac{7}{16}$″, double the smaller dimension, as it is off the scale, and halve the answer.

10. Values

The following sections on Values and Profit Margins are duplicated to cover calculations in (a) duodecimal sterling coinage and (b) any decimal coinage.

(a) Duodecimal Coinage

Scale C can be used to multiply or divide any figures set on Scale D. *For calculations involving shillings and pence, use Scale L, treating feet as shillings and inches as pence.*

For calculating the value of sq. yds. or any other units, set 1 on C over quantity on D, bring the cursor over the price per unit on C or L, and read the answer below it on D. If scale L has been used, the answer will be in shillings. To convert to pounds, bring 20 on C under the line and read the answer on D under 1 on C.

To find the value of 65 sq. yds. at 1/10½, set 1 on C over 6.5 on D, move the cursor to 1′ 10½″ on L and read the answer of 12.2x10 shillings as 122/– (121/9) on D under the line. To convert to pounds, move the slide so that 2 on C comes under the line and read the answer on D under 1 on C, i.e. 6.1 £, or say £6.2.0.

Scales L or C can also be used in conjunction with scales V and F. For instance, to find the value of 18.9 cu. yds. at 31/9 per cu. yd., set the cursor over 18.9 on V. This is near the end of the scale, so set 100 on L under the line and shift the cursor back to 31′ 9″ on L. The answer in shillings is now under the line on V, but to convert to pounds bring 2′ on L under the line, shift the cursor to 1′ on L and read £30 on V.

Remember that if the quantity is on V or F, the value is on V or F, not on D.

By using scale F in conjunction with scale D, any price per sq. yd. can be read off as a price per sq. ft. and vice versa.

To convert say 47/3 per sq. yd., set the cursor over 47.25 on F and read 5.25 on D above; that is, 5/3 per sq. ft. If in doubt about decimals of a shilling, set 1 on C over 1.2 on D. Then to check e.g. 0.47 shillings, set the cursor over 4.7 on C and read 5.64 pence, or rather more than 5½d. on D below. Conversely, 9½d. (9.5 on D) is shown as 0.79 shillings on C.

(b) Decimal Coinage

Scale C can be used to multiply or divide any figures set on scale D.

For calculating the value of sq. yds. or any other units, set 1 on C over the quantity on D, bring the cursor over the price per unit on C, and read the answer below it on D.

To find the value of 35 sq. yds. at $1.55 per sq. yd., set 1 on C over 35 on D, move cursor to 1.55 on C and read $54.4 on D. Alternatively, to calculate the price of 55 sq. yds. costing $76.50, set the cursor over 76.5 on D, bring 55 on C under the line and read $1.39 per sq. yd. on D under 1 on C.

Scale C can also be used in conjunction with scales V and F for multiplication or division. For instance, to find the value of 18.9 cu. yds. at $14.50 per cu. yd., set the cursor over 18.9 on V, bring 1 on C under the cursor line, move the cursor to 1.45 on C and read the answer as 27.4x10, or $274 on V.

Remember that if the quantity is on V or F, the value is on V or F, not D.

By using scale F in conjunction with scale D, any price per sq. yd. can be read off as a price per sq. ft. and vice versa. To convert $7.70 per sq. yd., set the cursor over 77 on F and read 8.56 on D above as 85.6 cents per sq. ft.

11. Profit Margins

A slide-rule is supreme when it comes to calculating percentages, for not only can a series of percentages be shown at a glance, but when comparing two values, mark-up and discount are shown simultaneously. For instance, set the cursor over 31 on D and bring 26 on C under the line. Now over 100 on D one can read that 26 is 84% of 31 (i.e. 31 less 16% discount) and under 1 on C one reads that 31 is 119% of 26 (i.e. 26 plus 19% mark-up).

(a) Duodecimal Coinage

Supposing that the estimated cost of a contract is £54.2.0 and you wish to add a gross margin of 27½%. Set the cursor over 54.1 on D (or preferably over 5.41 and multiply by 10 later) and bring 1 on C under the line. Now move cursor to 1.275 on C and read the grossed-up value of 69.0 £ on D. If a round sum of £70 is to be charged, set the cursor over 70 on D and read 1.29 on C, showing that the profit margin is now 29%. To convert this gross profit on cost into gross profit on selling price, set 1 on C over 1.29 on D and read 77.5 on C over 100 on D, showing a 22½% profit on selling price.

Supposing that the actual cost of the contract proved to be £56.10.0, set 56.5 on C over 70.0 on D and read 80.8 on C over 100 on D. This shows that only 19% instead of 22½% was achieved.

(b) Decimal Coinage

Supposing that the estimated cost of a contract is $541 and you wish to add a gross margin of 27½%. Set the cursor over 54.1 on D (or preferably over 5.41 and multiply by 10 later) and bring 1 on C under the line. Now move cursor to 1.275 on C and read the grossed-up value of $690 on D. If a round sum of $700 is to be charged, set the cursor over 70 on D and read 1.29 on C, showing that the profit margin is now 29%. To convert this gross profit on cost into gross profit on selling price, set 1 on C over 1.29 on D and read 77.5 on C over 100 on D, showing a 22½% profit on selling price.

Supposing that the actual cost of the contract proved to be $565, set 56.5 on C over 70.0 on D and read 80.8 on C over 100 on D. This shows that only 19% instead of 22½% was achieved.

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